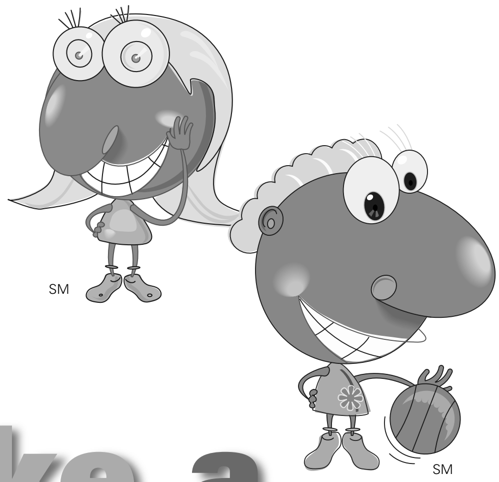
Figure This!
Math Challenges for Families



Take a Challenge!

Set I: Challenges 1 - 16

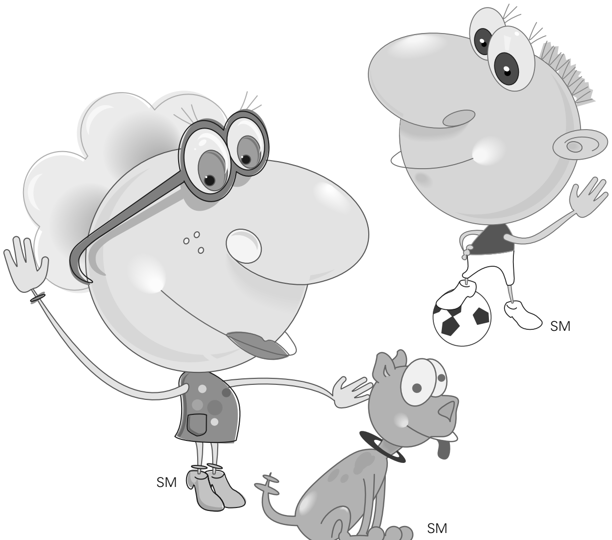
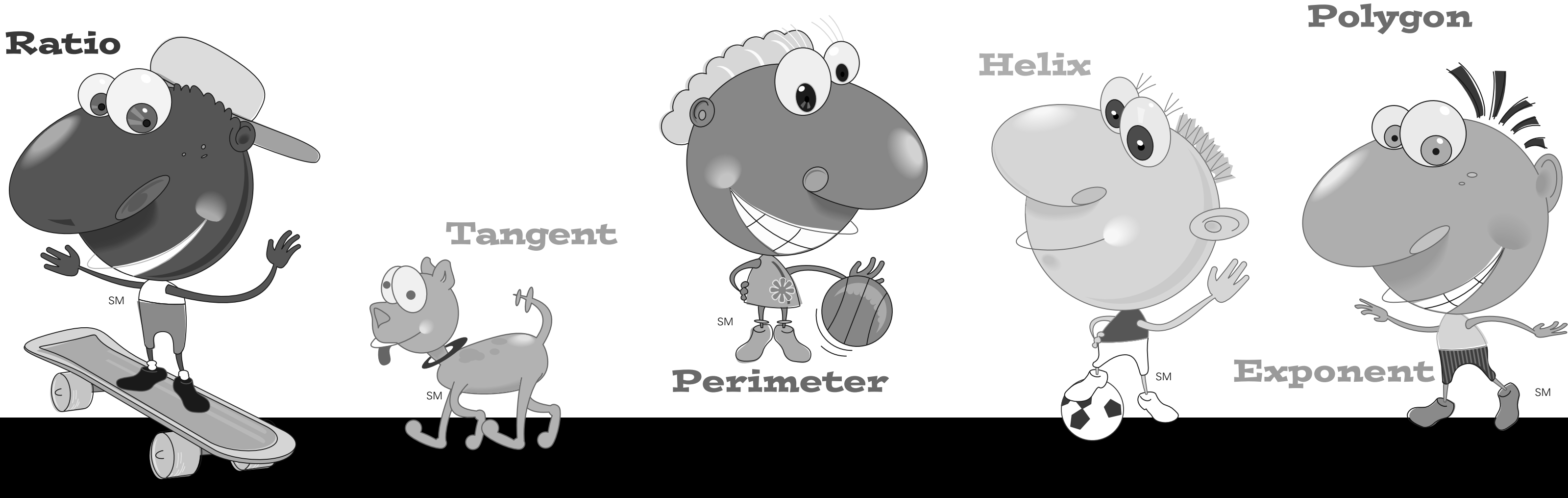


Figure This! Math Challenges for Families

Thank you for your interest in the **Figure This! Math Challenges for Families.** Enclosed please find Challenges 1 – 16. For information about other challenges, go to www.figurethis.org.

The **Figure This!** Challenges are family-friendly mathematics that demonstrate what middle-school students should be learning and emphasize the importance of high-quality math education for all students. This campaign was developed by the National Action Council for Minorities in Engineering, the National Council of Teachers of Mathematics, and Widmeyer Communications, through a grant from The National Science Foundation and the US Department of Education.



We encourage you to visit our website at **www.figurethis.org** where you can find these and other challenges, along with additional information, math resources, and tips for parents.

Figure This!

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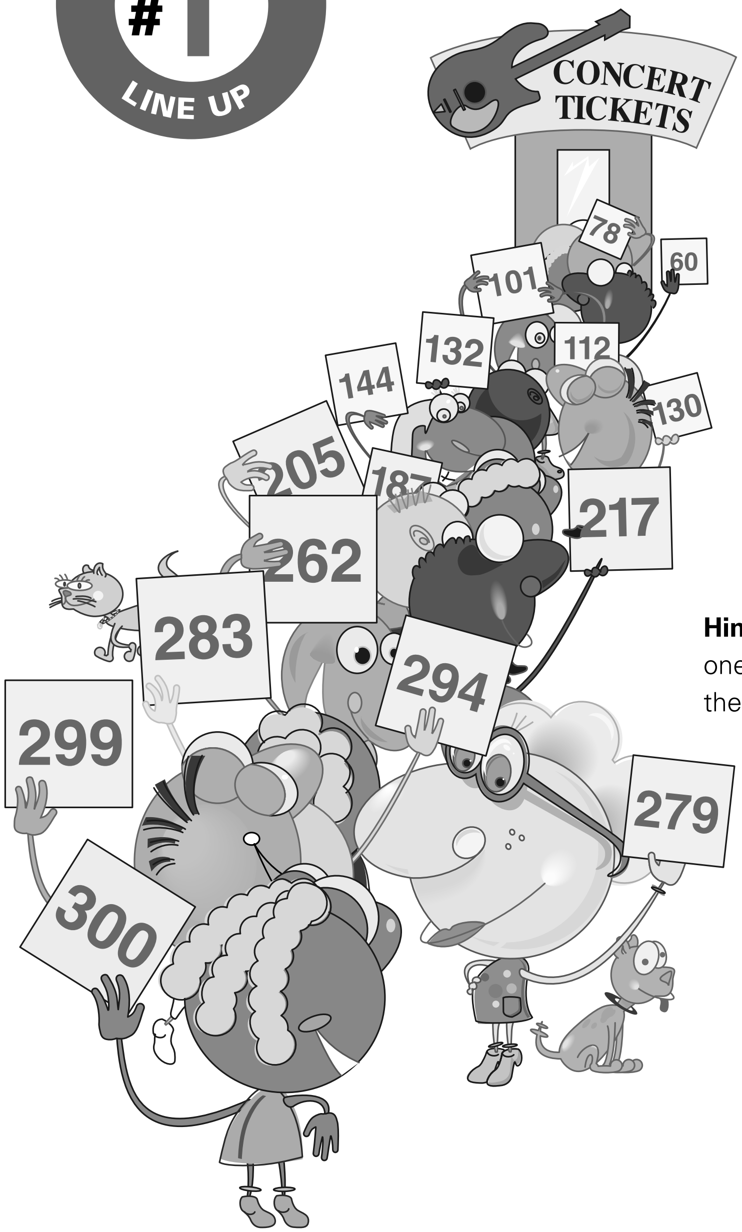
Math Challenge
#1
LINE UP

Figure This!
Math Challenges for Families



How long do you have to stand in line?

Figure This! **How long do you think you would have to wait in this line if you hold number 300?**

Hint: Estimate the amount of time it would take for one person to buy a ticket. Use this estimate to find the amount of time you will have to wait in line.

Estimation and measurement of time are basic skills for all. Businesses such as banks, fast-food restaurants, ski areas, and airports need efficient ways to minimize time spent waiting in line.

Answer:
Assuming that it takes about 30 seconds to buy a ticket, you would wait about 2 1/2 hours.

Figure This!

Get Started:

Pretend you are buying a ticket. How long would it take?

Complete Solution:

If each person takes 30 seconds to buy a ticket, then the 299 people in front will take about 30 x 299 or 8970 seconds about 150 minutes. That's 2 1/2 hours.

Another way to estimate this is to recognize that if it takes 30 seconds for one person to buy a ticket, then 2 people can buy a ticket in 1 minute. Thus, it takes 300 ÷ 2 or about 150 minutes for your turn to arrive. That's about a 2 1/2 hour wait.

Try This:

Go to a fast-food restaurant, supermarket, or someplace where people wait in line. Find the average number of people in line and the average amount of time they have to wait. Do different kinds of lines make a difference in waiting time? For example, is the express line really faster?

Additional Challenges:

1. If you expect 600 people to buy tickets to a rock concert at your school and the box-office opens one-half hour before the show, how many ticket sellers do you need?
2. If there are 300 people ahead of you in line to buy a ticket, about how many feet back are you?

Things to Think About:

- Do lines move faster at movies or at concerts?
- What factors would make the time for purchasing a ticket longer or shorter?

Did You Know That?

- The 1993 *Guinness Book of World Records* reported that the longest line of coins ever created was 34.57 miles long. It had 2,367,234 coins and was in Kuala Lumpur, Malaysia.
- The 1993 World Record for the most valuable line of coins was 1,724,000 quarters. The line was 25.9 miles long in Atlanta, GA.
- On April 21, 1990, an estimated 180,000 people in Maracana Stadium in Rio de Janeiro, Brazil, paid to hear Paul McCartney.
- At the London opening of *The Phantom Menace*, 11,500 tickets were sold in just 30 minutes on June 12, 1999.
- Queuing theory deals with wait times in lines.

Resources:

Book:

Matthews, Peter, ed. *The Guinness Book of World Records 1993.* New York: Bantam Books, 1993.

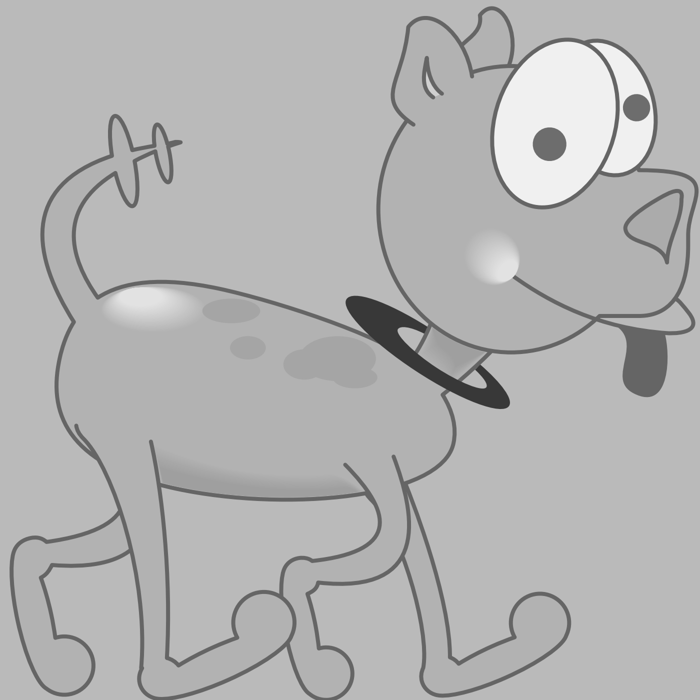
Website:

"Does this line ever move?" *Informs*
mie.eng.wayne.edu/faculty/chelst/informs

Answers to Additional Challenges:

(1.)
If each ticket purchase takes 30 seconds, it would take 5 hours for one person to sell 600 tickets. Since you only have one-half hour to sell the tickets, you would need 10 ticket sellers.

(2.)
If each person requires about 2 feet of space along the ground, then you are about 300 x 2, or 600 feet back in the line.



Math Challenge #2 BEATING HEART

FigureThis! Math Challenges for Families

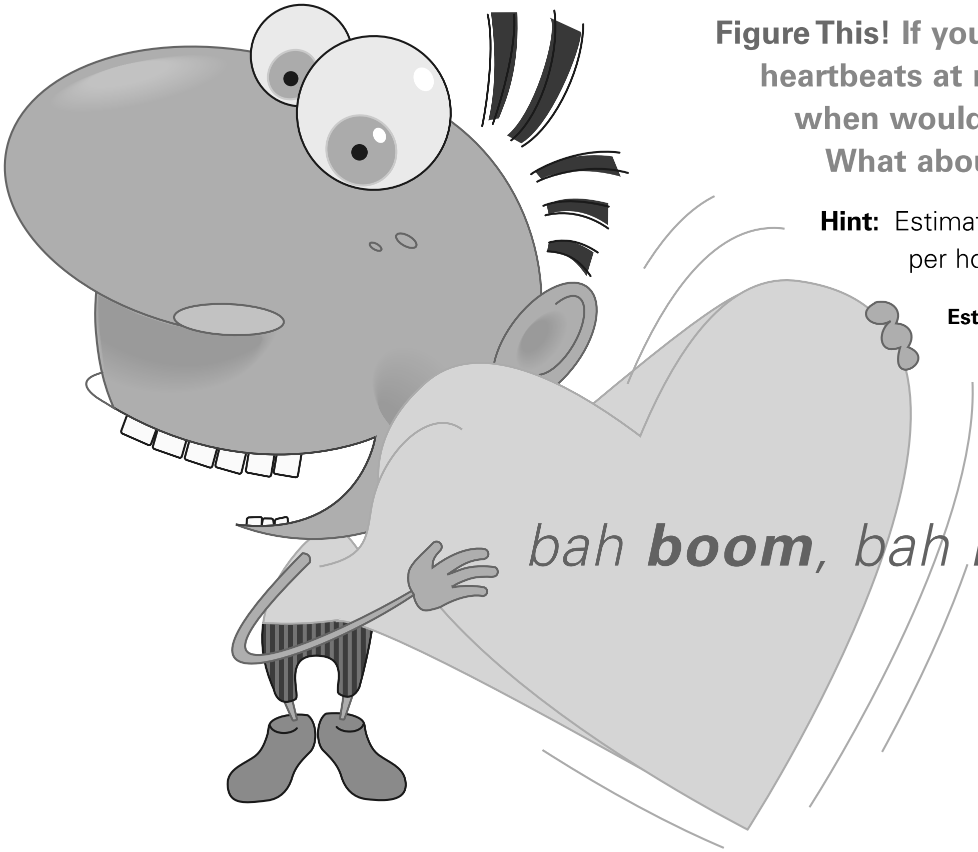
How *fast* does your heart beat?

How long does it take for your heart to beat 1000 times?

Figure This! If you started counting your heartbeats at midnight on January 1, 2000, when would you count the millionth beat? What about the billionth beat?

Hint: Estimate your heart rate in beats per minute, per hour, and per day.

Estimating and understanding large numbers are useful mathematical skills. Without these skills, it is difficult to comprehend the size of the national debt, for example, or how many miles it is to Mars.



bah ***boom****, bah* ***boom****, bah* ***boom***

Answer:

It takes about 15 minutes for a heart to beat 1,000 times. On January 10, 2000, your heart would have beat about 1 million times. 27 years later your heart would reach about 1 billion beats.

Figure This!

Get Started:

The best place to find your pulse is on your wrist or neck. Count the number of beats in your pulse for 15 seconds and multiply by 4, or count the number of beats for 30 seconds and multiply by 2. Once you know the number of times your heart beats in 1 minute, find the number of beats in 1 hour, and then in 1 day.

Complete Solution:

By estimating, you can often find a reasonable answer more quickly. Round your number of heartbeats to a number that is easy to use for calculation. For example, if your heart beats 72 times per minute, 70 beats per minute would give a reasonable estimate.

Beats per Hour	Beats per Day	Estimated Beats per Day
70 x 60 = 4200	4200 x 24	4000 x 25 = 100,000

Next find how many days it takes for 1 million heartbeats.

1,000,000 ÷ 100,000 = 10. Thus, 1,000,000 beats take 10 days.

If you begin counting on January 1, the millionth beat will be on January 10. For 1 billion heartbeats, the number of days is 1,000,000,000 ÷ 100,000 = 10,000. Since there are 365 days in a year, and 10,000 ÷ 365 is approximately 27.4, it will take about 27.4 years. The billionth beat will be in May 2027.

Try This:

Go to a website and search for large numbers using keywords such as billion, trillion, and large numbers. Try **www.mcn.net/~jimloy/trillion.html**

Additional Challenges:

1. Have you been alive for 1 million minutes? Do you know someone who has been alive for this long?
2. Have you been alive for 1 billion minutes? Do you know someone who has been alive for this long?
3. Have you been alive for 1 million hours? Do you know someone who has been alive for this long?
4. Suppose your heart rate was 72 beats per minute. You rounded it to 70 to make your calculation. If you did not round, how far off would your estimated answers be?

Things to Think About:

- If you had 1 million drops of water, would you be more likely to drink it, take a bath in it, or swim in it? What if you had 1 billion drops of water?
- There are approximately 3 million people in the city of Chicago. How many times larger or smaller is the population of your city than the population of Chicago?
- How much does rounding affect your answers when adding? When multiplying? How close is close enough?

Did You Know That?

- 1 million is 1000 times 1000, or $10^3 \cdot 10^3 = 10^6$, no matter where you live, but the definition of 1 billion depends on where you are in the world.
- In the USA, 1 billion is 1000 times 1 million, or, $10^3 \cdot 10^6 = 10^9$, but in Great Britain and France 1 billion is 1 million times 1 million, or $10^6 \cdot 10^6 = 10^{12}$

Resources:

Books:

- Morrison, Philip. *Powers of Ten.* New York: Scientific American Books, W. H. Freeman, 1982.
- Paulos, John Allen. *Innumeracy*. New York: Hill and Wang, 1988.
- Schwartz, David M. *How Much Is a Million?* New York: William Morrow & Company, 1994.
- Schwartz, David M. *If You Made a Million.* New York: William Morrow & Company, 1994.
- Strauss, Stephen. *The Sizesaurus*. New York: Avon Books, 1997.

Answers to Additional Challenges:

(1.)
Yes. There are 60 minutes in an hour, 24 hours in a day, and 7 days in a week. Since 60 x 24 x 7 = 10,080 there are close to 10,000 minutes in a week. Thus, 1 million minutes is about 100 weeks, or about 2 years, so anyone more than 2 years old has been alive for at least 1 million minutes.

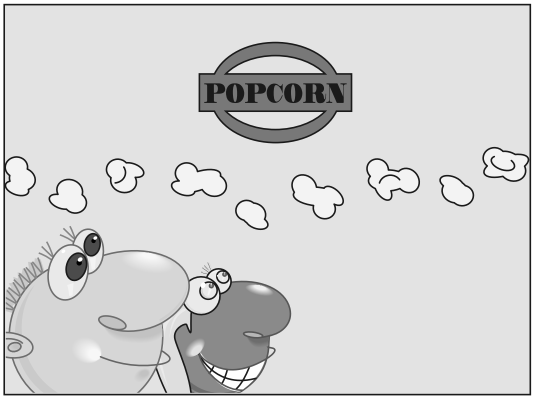
(2.)
No. Since 1 billion is a thousand million, 1 billion minutes is around 1000 x 2 , or 2000 years.

(3.)
There are approximately 24 x 365, or 8760 hours in a year. Because 1,000,000÷8760 is a little more than 114, it is unlikely that you know anyone who has been alive for 1 million hours, about 114 years.

(4.)
For 1 million beats, the answer is off about 15.5 hours. For 1 billion beats, the answer is off about 9 months.

Math Challenge #3 POPCORN

FigureThis! Math Challenges for Families

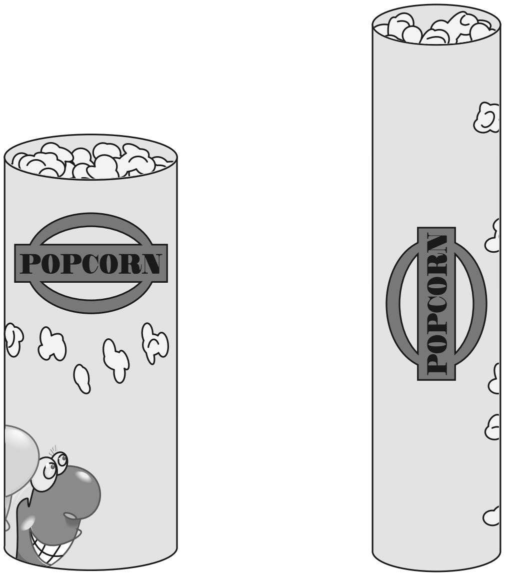


If you like popcorn, which one would you buy?

Figure This! Take two identical sheets of paper. [An ordinary sheet of paper measures 8 1/2 inches by 11 inches.] Roll one sheet into a short cylinder and the other into a tall cylinder. Set them both on a flat surface. Does one hold more than the other?

Hint: Place the taller cylinder inside the shorter one. Fill the taller one with dry cereal, rice, or popcorn; then remove it from the shorter cylinder. Which holds more?

Making visual estimates and finding volumes are useful skills. Designers and engineers use these skills to find economical ways to package and protect items.



Answer:
The shorter cylinder holds more.

Figure This!

Get Started:

Make a guess and then use the hint.

Complete Solution:

The process described in the "Hint" shows that the shorter cylinder holds more.

- To determine an answer mathematically, find the volume of each cylinder. The volume is the area of the base times the height. In this case, the bases are circular. The area of a circle is $\pi \cdot r \cdot r$ or approximately $3.14 \cdot r \cdot r$. To find the radius, r, use a ruler to estimate the width (or diameter) of the circle. Divide the diameter by 2 to get the radius. Another way to find the radius of a circle is to use the formula:

  Circumference of circle = $2 \cdot \pi \cdot$ radius = $2\pi r$
  Radius = Circumference divided by (2π)

  Once you have the radius, the table below shows how to determine the volume of each cylinder. The sheet of paper is 8 1/2 inches by 11 inches.

Cylinder	Base (inches) Circumference	Radius (inches) r	Height (inches) h	Volume (cubic inches) $\pi \cdot r \cdot r \cdot h$
Short	11	$11 \div (2\pi)$ or about 1.75	8.5	$\pi \cdot 1.75 \cdot 1.75 \cdot 8.5$ About 81.8
Tall	8 1/2= 8.5	$8.5 \div (2\pi)$ or about 1.35	11	$\pi \cdot 1.35 \cdot 1.35 \cdot 11$ About 63.0

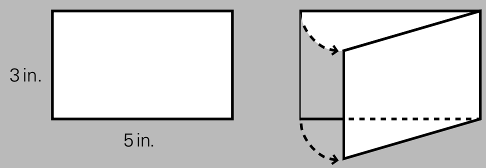
The volume of the shorter cylinder is about 82 cubic inches, and the volume of the taller cylinder is about 63 cubic inches.

Try This:

- Go to the grocery store and see what items come in different shaped or different sized cylinders.
- Look in your cupboard or go to the grocery. Find two different shaped containers that hold the same amount. What are the contents of each?

Additional Challenges:

1. For what size paper would the two cylinders hold the same amount?
2. Using any non-square rectangular sheet of paper, does the shorter cylinder always hold more?
3. Another way to describe a cylinder is to rotate an index card about one of its sides. Think about the cylinder traced by a 3 by 5 card as it turns. Which is larger: the volume of the cylinder formed when the card is turned about a short side or a long side?



4. Suppose you had two equal lengths of wire. Fold the wires to make two rectangles. Do you think the two rectangles will always have the same area?

Things to Think About:

- What items come packaged in cylindrical containers?
- What types of goods are packaged in boxes instead of cylinders? Why do you think companies use boxes?
- If a number is greater than 1, squaring it makes the result greater.

Did You Know That?

- Designing cans and labels is just one aspect of packaging technology. You see the results of this work every time you unwrap a CD, twist open a lipstick container, or open a soft drink.
- Several universities offer degrees in packaging technology. Many of them can be found at **packaging.hp.com/pkgschools.htm**
- Isopermetric figures are figures with the same perimeter. Fencing problems typically fall under this category.

Resources:
Books:

- Lawrence Hall of Science. *Equals Investigations: Flea-Sized Surgeons*. Alsip, IL: Creative Publications, 1994.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and E. Phillips. *Connected Mathematics: Filling and Wrapping*. Palo Alto, CA: Dale Seymour Publications, 1996.

Answers to Additional Challenges:

(1.)
The two cylinders would hold the same amount only for square sheets of paper.

(2.)
Yes. (This can be proven mathematically.)

(3.)
The cylinder with the larger volume is described when the card is turned about its shorter side. This problem compares $\pi \cdot 5 \cdot 5 \cdot 3$ and $\pi \cdot 3 \cdot 3 \cdot 5$.

(4.)
The rectangles may not always have the same area. Consider a piece of wire 16 inches long. You can make a square 4 inches by 4 inches with an area of 16 square inches, or a 1 inch by 7 inches rectangle that has an area of 7 square inches.

Math Challenge #4 DON'T FALL IN!

FigureThis! Math Challenges for Families

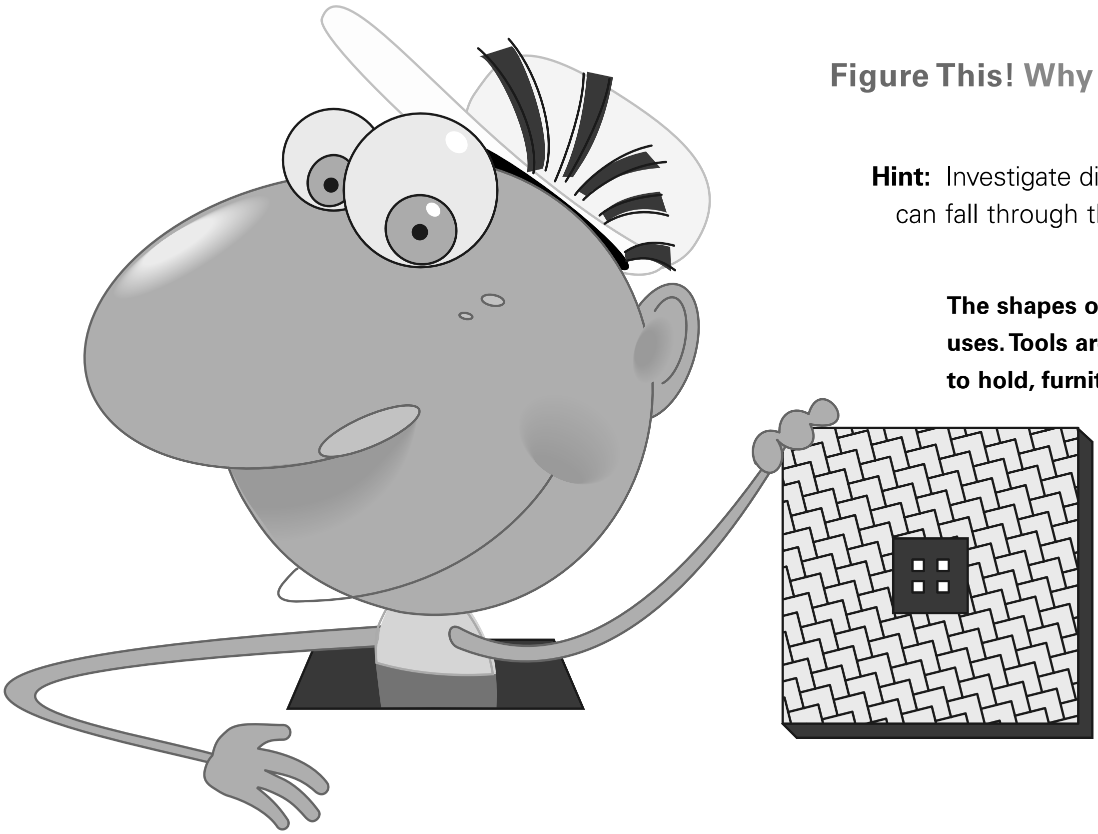
Why?

Why aren't manhole covers square?

Figure This! Why are most manhole covers round?

Hint: Investigate different shaped covers to see if they can fall through their corresponding holes.

The shapes of many objects relate directly to their uses. Tools are designed with shapes that are easy to hold, furniture is designed with shapes that are comfortable, and race cars are designed to reduce wind resistance.



Answer:
Square manhole covers can be tipped diagonally and fall through the hole.

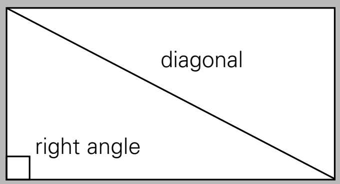
Figure This!

Get Started:

Cut a rectangle and a circle out of a 3 by 5 card or light cardboard. See which will easily fall through the hole left in the cardboard.

Complete Solution:

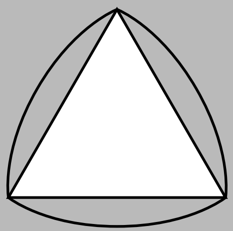
A manhole cover rests on a small lip inside the hole. A circular manhole cover typically will not fall into the hole because its width is the same all around. A rectangular manhole cover, however, could fall through the hole when it is tipped upward. You can see this by drawing a diagonal in a rectangle to create two triangles. Mathematically, the greatest angle of a triangle is opposite the longest side. The greatest angle in each triangle formed by drawing in the diagonal is the right angle at the corner.



This means that the diagonal of a rectangle is always longer than either of the sides. As a result, rectangular covers can always be dropped through their corresponding holes if the lips are small. Because a square is a rectangle, the same reasoning applies to squares.

Try This:

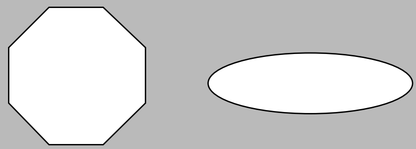
- Look around you and find some triangles. Decide which angle is the largest and check to see if it is opposite the longest side. Cut a triangle from a piece of cardboard and see if it can easily fall through the hole formed where you cut it out.
- On a card, draw a triangle with all sides the same length (an equilateral triangle). Next, draw arcs around the triangle, using each corner as the center of a circle and the length of a side of the triangle as the radius. (See the diagram below.)



Cut out the shape and see whether it would fall through the hole left in the card. Measure from any point of the shape to the other side. What do you notice about the widths? With the usual small lip of a manhole, would this shape make a good cover?

Additional Challenges:

1. Would the following shapes make good manhole covers?
2. Name a three-dimensional shape that has the same width all around.



Things to Think About:

- A circle is known as a "curve of constant width." Why do you suppose that such curves have this name? Can you think of any other shape that has a constant width? Does the figure you made in the "Try This" section above have a constant width?
- Will any manhole cover that is not a curve of constant width fall into its hole?

Did You Know That?

- A drill bit based on the triangular shape in the "Try This" section cuts a square hole.
- The bases of Pepto-Bismol™ bottles have shapes like the one in the "Try This" section.
- Triangular manhole covers are used in some places in Minnesota.
- Curves of constant width were studied by the Swiss mathematician, Leonhard Euler (1707-1783).
- Reuleaux triangles were named for Franz Reuleaux (1829-1905), a German engineer, mathematician and high school teacher.

Resources:
Books:

- Gardner, Martin. *Mathematics, Its Spirit and Use*. New York: W. H. Freeman, 1978.
- Maletsky, Evan. "Curves of Constant Width." In *Teaching with Student Math Notes, Vol. 2*. Reston, VA: National Council of Teachers of Mathematics, 1989.
- Melnick, Mimi. *Manhole Covers*. Cambridge, MA: MIT Press, 1994.

Answers to Additional Challenges:

(1.) Neither would make a good cover because their widths are not the same all around.

(2.) A sphere.

Math Challenge #5 UPSIDE DOWN?

Figure This! Math Challenges for Families

How would you hang this sign

Figure This! **What letters, when written in lowercase, can be read the same upside down as right side up?**

Hint: Write out each lowercase letter and look at it in different ways.

Symmetry is a basic geometric concept. Understanding how one part of an object mirrors the rest is important in art, design, medicine and other fields.

Answer:

Depending on the way the letters are written or printed, l, o, s, x and z can be read the same upside down and right side up.

Figure This!

Get Started:

Print the lowercase letters of the English alphabet. Turn your paper upside down.

Complete Solution:

Letters that read the same upside down are l, o, s, x, and z.

Try This:

- Distort the letters a bit and see if you can write your name so that it can be read either backward or forward or upside down. For example, the words "upside down" above are distorted so that they can be read when turned halfway around.
- Look around and see if you can find shapes that look the same backward and forward or upside down and right side up.
- Look in the Yellow Pages to find logos of companies that are the same when looked at from different directions.

Additional Challenges:

1. When written in lowercase letters, the name of one professional sports team can be read the same both up right and upside down. What team is it?
2. Create words that are spelled the same backward or forward. Such words are palindromes.
3. Create sentences that read the same backward or forward when punctuation is ignored.
4. What times on a digital clock can be read the same in different directions?
5. Some letters can be rotated 180˚ (or a half circle) to form different letters; for example "d" becomes "p." What other lowercase letters can be rotated to form different letters?

Things to Think About:

- Does a human figure have symmetry?
- How is symmetry used in the design of a pinwheel?
- Is there symmetry in nature?

Try This:

Find the symmetry, if any, in each of the following:

- a plate
- a bowl
- a fork
- a chair

Did You Know That?

- Scott Kim calls anything that can be read in more than one way an "inversion." He used such writing in developing new font systems for computers.
- Graphic artist John Langdon of Philadelphia, has developed writing similar to Kim's that he calls "ambigrams."
- Artist M. C. Escher used many types of symmetries in designing his famous tessellations.
- A circle and a sphere have the most symmetries of any geometric objects.
- A figure with both horizontal and vertical lines of symmetry also has 180˚ rotational symmetry, but not the other way around.

Resources:

Books:

- Ernst, Bruno. *The Magic Mirror of M. C. Escher.* Stradbroke, England: Tarquin Publications, 1985.
- Kim, Scott. Inversions: *A Catalog of Calligraphic Cartwheels*. Petersborough, NH: BYTE Books, 1981.
- Kim, Scott. *Poster: Alphabet Symmetry*. White Plains, NY: Cuisenaire/Dale Seymour Publications. **www.cuisenaire-dsp.com**
- Kim, Scott. *Poster Set: Inversions*. White Plains, NY: Cuisenaire/Dale Seymour Publications. **www.cuisenaire-dsp.com**
- Langdon, John. *Wordplay: Ambigrams and Reflections on the Art of Ambigrams.* New York: Harcourt Brace Jovanovich, 1992.
- McKim, Robert H. *Experiences in Visual Thinking*. Belmont, CA: Brooks/Cole Publishing Company, 1972.
- McKim, Robert. *Thinking Visually*. White Plains, NY Cuisenaire/Dale Seymour Publications, 1997. **www.cuisenaire-dsp.com**

Credits:

Answers to Additional Challenges:

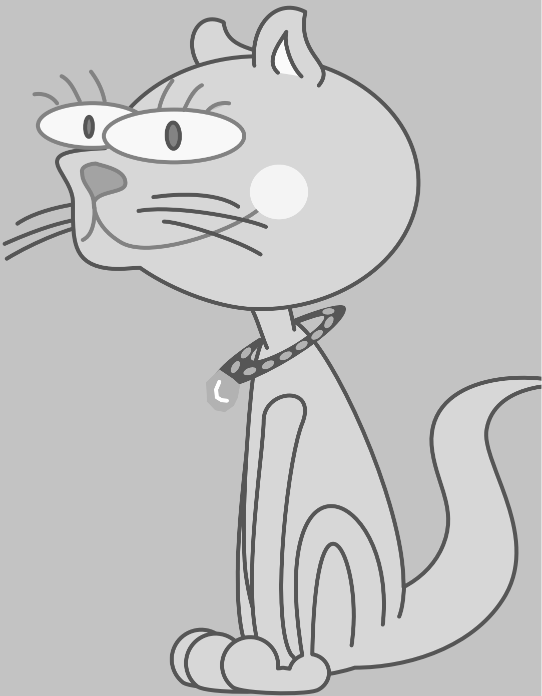
(1.)
Phoenix Suns. The word "suns" reads the same upside down.

(2.)
Words like MOM, DAD, and TOOT qualify. The international distress symbol, SOS, is a set of letters that can be read the same backward, forward, up right, and upside down as well as when turned halfway around.

(3.)
"Madam I'm Adam."

(4.)
Numbers such as 0, 1, 2 and 8 read the same in several ways on a digital clock. Times on a digital clock that read the same backward as forward include 10:01, 11:11, and 12:21.

(5.)
The letter "m" becomes "w," "n" becomes "u," and "b" becomes "q."



Math Challenge #6 WHICH WAY?

Figure This! Math Challenges for Families

Oh, which way do I go?

How can you go straight to the store when there are buildings in the way?

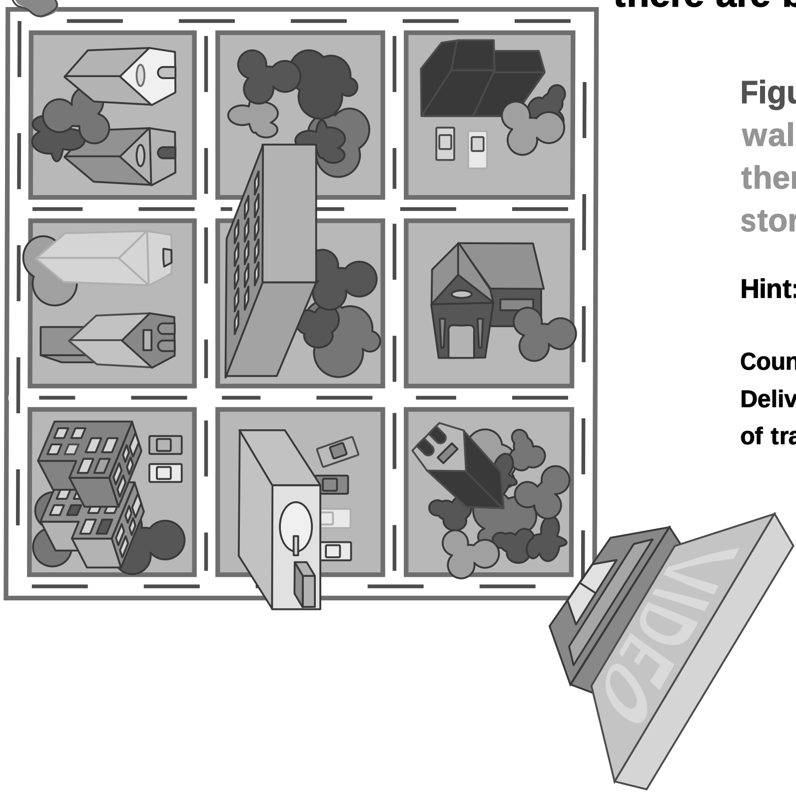


Figure This! By walking on the sidewalk, how many different ways are there to go from home to the video store? No backtracking allowed!

Hint: Try fewer blocks to start.

Counting is an important mathematical skill. Delivery companies and airlines count the number of travel routes to get from one place to another.

Answer:

There are 20 different ways to get from home to the store.

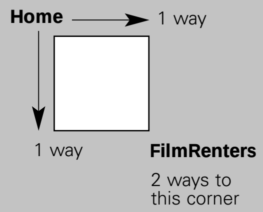
Figure This!

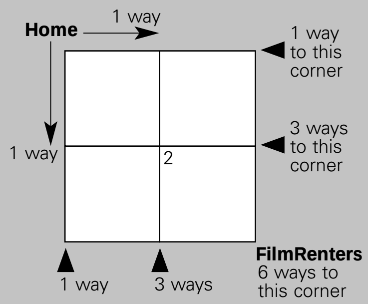
Get Started:

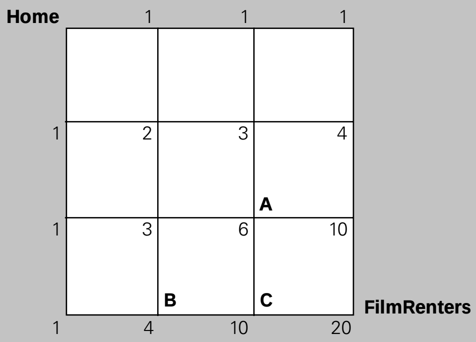
How many ways would there be if your home and the store were on opposite corners of the same block? If the store were two blocks away? Do you have to count both the paths that start south and east separately?

Complete Solution:

Look at a simpler case and count the number of ways to each corner. In the drawings below for example, the arrows have been added to show direction. The numbers indicate how many ways there are to get to each corner.







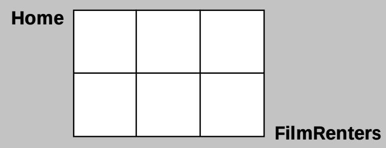
To get to corner C, you must pass through either corner A or B. There are 6 ways to get to corner A, and 4 ways to get to corner B making a total of 10 ways to get to corner C. Thus, there are 20 ways to get from home to FilmRenters.

Try This:

- Find the shortest route from your home to school. Are there different routes of the same length?

Additional Challenges:

1. How long is the shortest route from home to FilmRenters in the Challenge?
2. How many ways are there to go from home to FilmRenters on the map below?

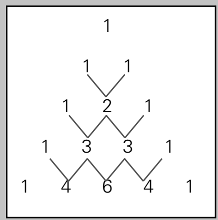


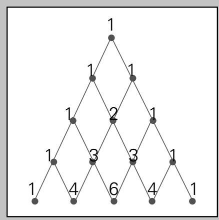
Things to Think About:

- Why is choosing efficient routes important to companies involved in transportation?
- What other jobs involve choosing efficient routes?

Did You Know That?:

- Blaise Pascal was a French mathematician in the 1600s. He worked with a pattern of numbers (Pascal's triangle) to solve many counting problems. Pascal's triangle is formed putting 1s along two "sides" of a triangle, then adding the two numbers above to the right and left to get the next number in the pattern.





- Pascal's triangle can be used to solve the challenge.
- Counting with patterns using Pascal's triangle was in Chu Shih-chieh's *Precious Mirror of Four Elements*, a fourteenth century book in China.
- Combinatorial analysis is a branch of mathematics that deals with counting problems like the one in this challenge.

Resources:

Books:

- Gardner, Martin. "Pascal's Triangle" in *Mathematical Carnival.* Washington, D.C.: Mathematical Association of America, 1989.
- Seymour, Dale, and Margaret Shedd. *Finite Differences*. White Plains, NY: Dale Seymour Publications, 1997. **www.cuisenaire-dsp.com**
- Seymour, Dale. *Visual Patterns in Pascal's Triangle.* White Plains, NY: Dale Seymour Publications, 1986. **www.cuisenaire-dsp.com**

Website:

www.studyweb.com/math

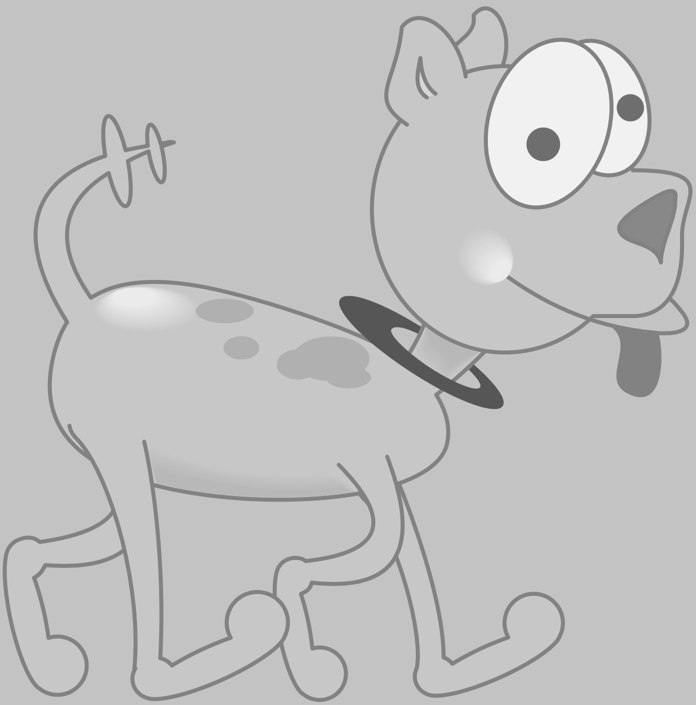
Answers to Additional Challenges:

(1.)

All 20 routes have the same length, 6 blocks.

(2.)

There are 10 ways to go.

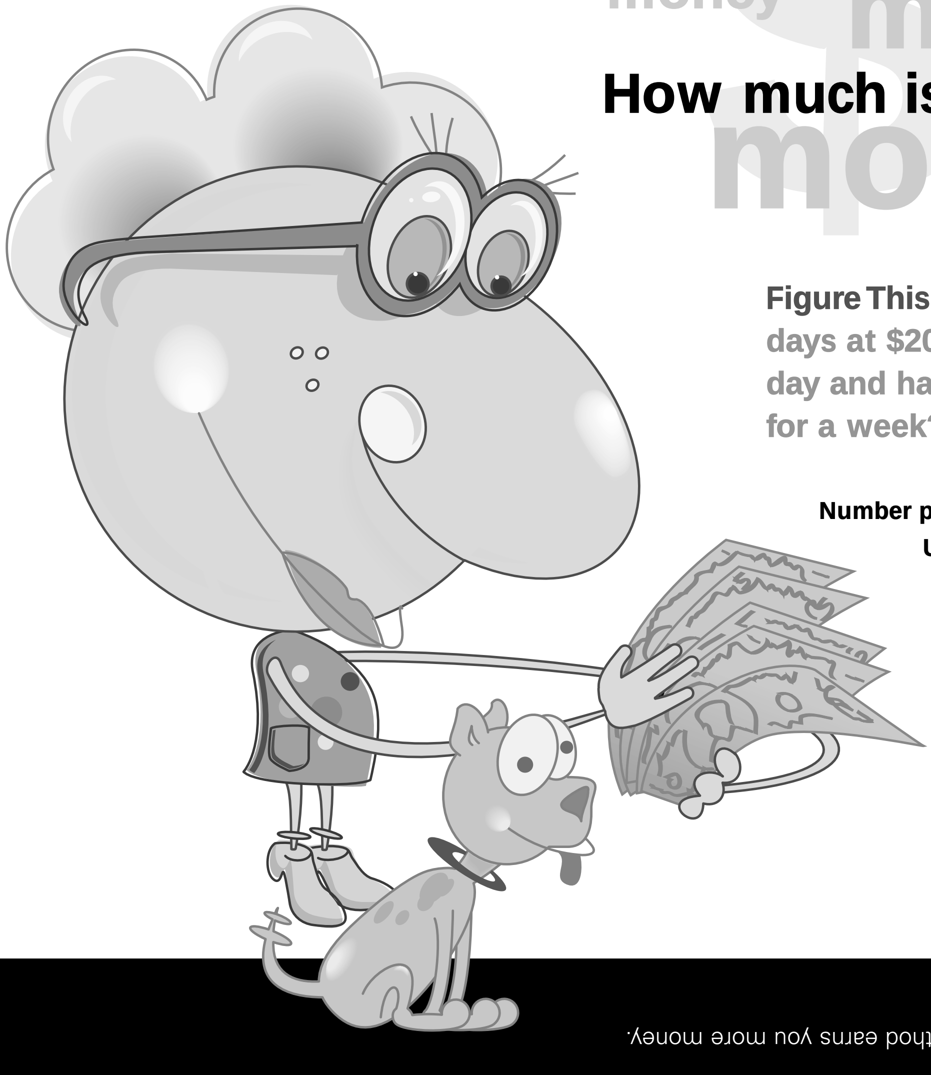


Math Challenge #7
DOUBLE OR NOT

FigureThis!
Math Challenges for Families

How much is your time worth?

Figure This! Would you rather work seven days at $20 per day or be paid $2 for the first day and have your salary double every day for a week?



Number patterns can change at very different rates. Understanding rates of change is important in banking, biology, and economics.

Answer:

If you are working the entire week, the doubling method earns you more money.

Figure This!

Get Started:

How much would you earn the first day using each method? The second day? What would be your total earnings at the end of the second day?

Complete Solution:

If you are paid $20 per day for seven days, then you earn $20 x 7 or $140. If you are paid $2 the first day and your salary doubles every day for the next six days, then you earn $2 + $4 + $8 + $16 + $32 + $64 + $128, or $254. The second scheme earns you more money by the end of the week.

Try This:

- Take a piece of paper of any size. Fold it in half. How does the thickness of the folded paper compare to the thickness of the unfolded paper? Repeat this process, answering the question each time. How many times can you fold the paper? Does the size of the paper with which you started make any difference? How thick is the folded paper when you can no longer fold it?
- Use a calculator to skip count by 3s. In other words, make the calculator show the sequence 3, 6, 9, 12, …. Can you make it multiply by 3s?

Additional Challenges:

1. If the payment methods described in the "Challenge" were carried out for a month, how much money would you have earned altogether on the 30th day?

2. What process is used to generate each of the following patterns?

- Cup, pint, quart, half-gallon, and gallon
- Penny, dime, dollar, ten dollars, one hundred dollars, and so on.

Things to Think About:

- How does the amount of money in a savings account increase?
- How do banks determine what interest is paid before the principal on loans?

Did You Know That?

- Radioactivity is often described by a half-life, the time required for half the radioactive material to decay.
- Counting by 1s is an example of an arithmetic sequence.
- A sequence like 2, 5, 8, 11, …, in which you add 3 each time, is also an arithmetic sequence.
- A sequence like 2, 6, 18, 54, …, in which you multiply by 3 each time, is a geometric sequence.
- The graph of an arithmetic sequence lies along a straight line.

Resources:

Books:

- Page, D., K. Chval, and P. Wagreich. *Maneuvers with Number Patterns*. White Plains, NY: Cuisenaire/Dale Seymour Publications, 1994. **www.cuisenaire.com**
- Seymour, Dale, and Ed Beardslee. *Critical Thinking Activities in Patterns, Imagery and Logic*. Vernon Hills, NY: ETA 1997. **www.etauniverse.com**

Software:

- "Bounce" (computer software package). Pleasantville, NY: Sunburst Communications, 1999. **www.sunburstdirect.com**

Answers to Additional Challenges:

(1.)

$600 by the first method and $2,147,483,646 by the second.

(2.)

In terms of cups, the pattern is 1 cup, 2 cups, 4 cups, 8 cups, 16 cups. Each measure is twice the previous one. In the sequence of money, each amount is 10 times the previous one giving 1 penny, 10 pennies, 100 pennies, and so on.

Math Challenge

#8

STAMPS

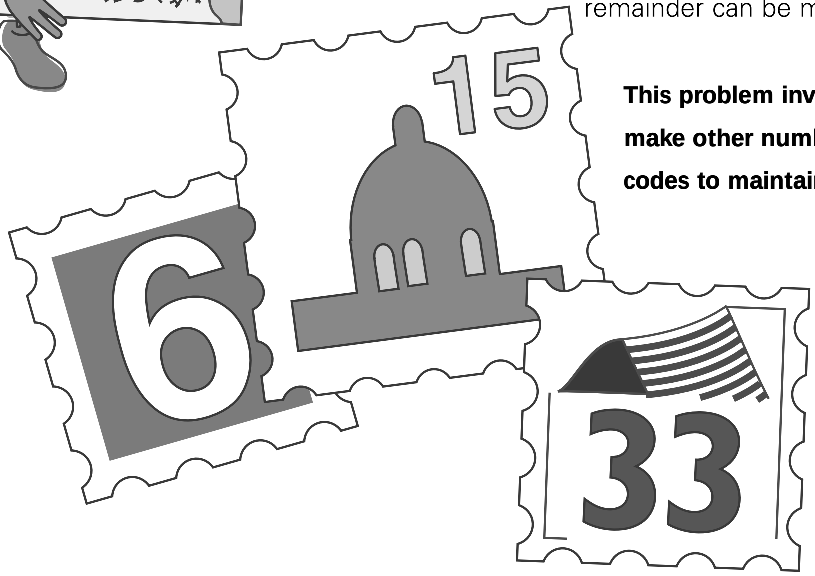
Figure This!

Math Challenges for Families

How can you use OLD stamps???

Figure This! **Suppose you found an old roll of 15¢ stamps. Can you use a combination of 33¢ stamps and 15¢ stamps to mail a package for exactly $1.77?**

Hint: Use as many 33¢ stamps as you can so that the remainder can be made with 15¢ stamps.



This problem involves using combinations of numbers to make other numbers. Similar processes are used to develop codes to maintain security in banking and computer access.

Answer:
Four 33¢ stamps and three 15¢ stamps equals $1.77 in postage.

Figure This!

Get Started:

What would happen if you tried to use only 33¢ stamps or only 15¢ stamps? What is the closest postage you can get if you use only 33¢ stamps?

Complete Solution:

There are many ways to do the problem.

- Using the hint given above, divide 177 by 33. With five 33¢ stamps, you have $1.65 worth of postage and need 12 more cents. With four 33¢ stamps, you have $1.32 worth of postage and need 45 more cents. Since three 15¢ stamps make 45¢, one solution is four 33¢ stamps and three 15¢ stamps.

- Another strategy is to make a table listing all of the possible combinations of 33¢ and 15¢ stamps that can be used.

Number of 33¢ Stamps	Number of 15¢ Stamps	Value
6	0	$1.98
5	0	$1.65
5	1	$1.80
4	1	$1.47
4	2	$1.62
4	3	$1.77

- Make a combination chart and fill it out until you either reach or pass $1.77 in every row and column. The first row has the values of an increasing number of 33¢ stamps and the first column the values of increasing the number of 15¢ stamps. Remaining cells are the combinations of the two.

		Number Of 33¢ Stamps					
		1	**2**	**3**	**4**	**5**	**6**
Number of 15¢ Stamps		$.33	$.66	$.99	$1.32	$1.65	$1.98
1	$.15	$.48	$.81	$1.41	$1.47	$1.80	
2	$.30	$.63	$.96	$1.29	$1.62	$1.95	
3	$.45	$.78	$1.11	$1.44	**$1.77**		
4	$.60	$.93	$1.26	$1.59	$1.92		
5	$.75	$1.08	$1.41	$1.74	$2.07		
6	$.90	$1.23	$1.56	$1.89			
7	$1.05	$1.38	$1.71	$2.04			
8	$1.20	$1.53	$1.86				
9	$1.35	$1.63	$2.01				
10	$1.50	$1.83					
11	$1.65	$1.98					
12	$1.80						

- Another way to approach the problem is to realize that using only 15¢ stamps makes the total end in either 0 or 5. Now look at the number of 33¢ stamps that can be used to get a total that ends in either 2 or 7.

Try This:

- Compare the cheapest way of mailing a package or letter using the U.S. Postal Service, Federal Express, or United Parcel Service.

Additional Challenges:

1. Using only 33¢ and 15¢ stamps, can you make any of the following?

a. $2.77 b. $4.77 c. $17.76

2. Can every whole number greater than 1 be made by adding some combination of twos and threes?

Things to Think About:

- Airmail stamps, which cost more than first-class postage, were once very popular for domestic mail. Why do you think this is no longer true today?

Did You Know That?

- The postage for a first-class stamp on July 6, 1932, was 3¢. It remained at that price until August 1, 1958, when it rose to 4¢.
- The price of a first-class stamp changed twice in 1981, to 18¢ in March and to 20¢ in November.
- More than 400 commemorative 32¢ stamps were issued from January 1, 1995 through December 31, 1998.
- On March 3, 1997, two triangular stamps depicting a clipper ship and a stagecoach were issued.
- Diophantus (ca. 250) is known as the Father of Algebra. Equations of the type in this challenge are called diophantine equations.
- The Euclidean Algorithm can be used in solving this challenge.

Resources:

Books:

- *New York Times World Almanac and Book of Facts 1999*, New York: World Almanac Books, 1999.
- "Media Clips." *Mathematics Teacher 92* (April 1999): 336-338.

Websites:

- U.S. Post Office **www.usps.gov**
- National Postal Museum **www.si.edu/postal**
- Foreign Postal Sites **www.upu.int/web/An**

Answers to Additional Challenges:

(1.)

These Challenges encourage solutions based on the reasoning in the original challenge.

a. No. There is no way to make the additional dollar of postage from combinations of 33¢ and 15¢ stamps. (Note that 100 is not a multiple of 3 as are 33 and 15. Also, there are no other combinations that work.)

b. Yes. You already know how to make $1.77 in postage. Make the additional $3.00 by using five 33¢ stamps and nine 15¢ stamps, or twenty 15¢ stamps. Either twelve 15¢ stamps and nine 33¢ stamps or twenty-three 15¢ stamps and four 33¢ stamps make $4.77 in postage.

c. Yes. You already know how to make $1.77 and $3.00 in postage. Make increments of $3.00 using twenty 15¢ stamps. Five sets of $3.00 plus the set for $1.77 equals $16.77. Since $17.76 is 99¢ more than $16.77, you need three more 33¢ stamps. The $17.76 in postage can be made using thirty-two 33¢ stamps and forty-eight 15¢ stamps, or one hundred three 15¢ stamps and seven 33¢ stamps.

(2.)

Yes.

Math Challenge #9 MIRROR, MIRROR...

FigureThis! Math Challenges for Families

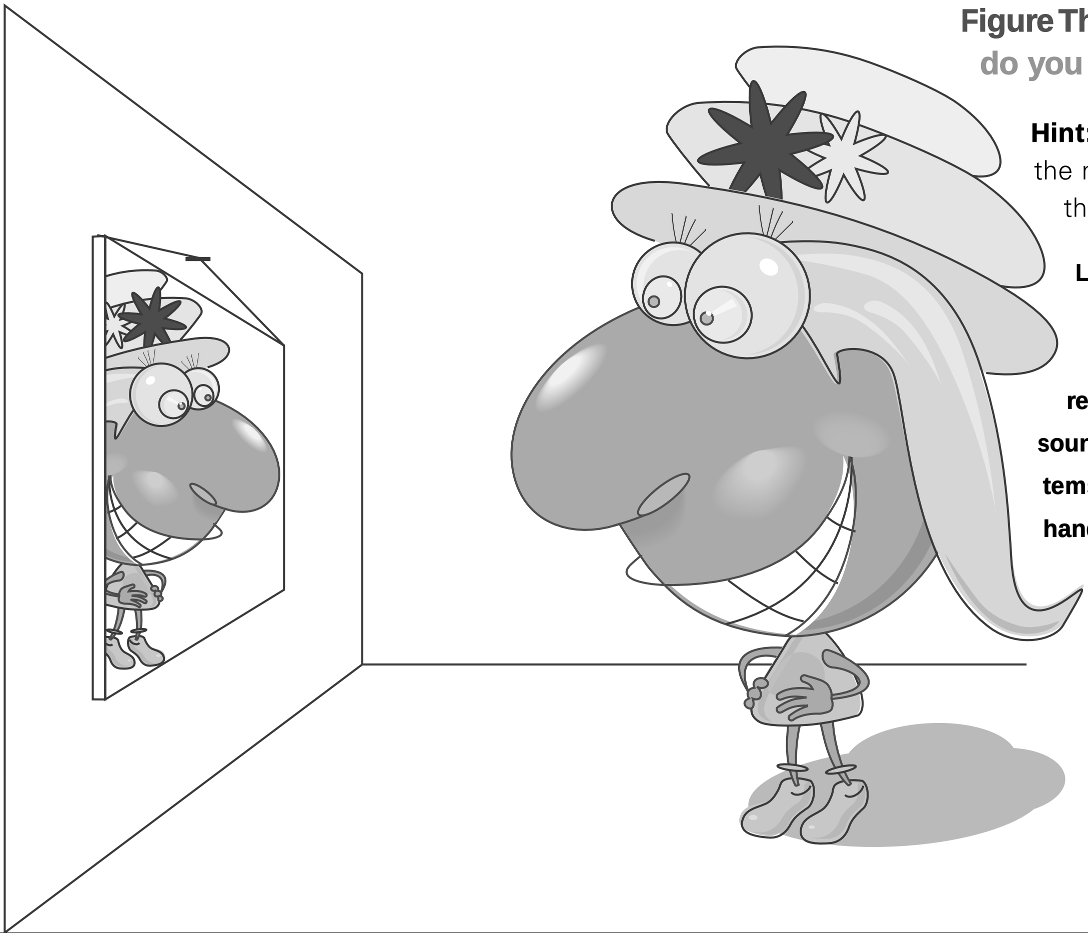
Oh! mirror, mirror

"Mirror, mirror, what do I see? Does backing up show more of me?"

Figure This! How much of yourself do you see in a small mirror?

Hint: Begin by measuring the height of the mirror and the amount of yourself that you can see.

Looking in mirrors involves angles, reflections, lines of sight, and triangles. Understanding how these are related is important in the design of sound stages, theatres, and security systems. Such knowledge can also come in handy when playing billiards, racquetball, tennis and some video games.



Answer:

When you look in the mirror and then back up, you see exactly the same amount of yourself.

Figure This!

Get Started:

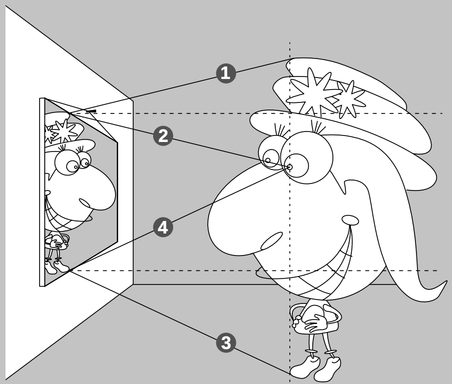
Start by standing about 2 feet from a wall. Have a friend hold a small mirror against the wall so that you can see only a portion of yourself; back up and see if you can see more of yourself.

Challenge:

Measure the height of the mirror. Measure how much of yourself you see in this mirror (the uppermost part to the lowermost part you see). Compare the measurements. Try other mirrors. Stand in different places. Draw a picture. Put the measurements in your picture. What do you notice?

Complete Solution:

The drawing shows that you see twice as much of yourself as the length of the mirror. Suppose that the mirror is hung flat against the wall so that the top of the mirror is halfway between your eye and the top of your hat. Lines 1 and 2 show the "line of sight" from your eye to the mirror and from the mirror to the top of your hat. Lines 3 and 4 show the "line of sight" from your eye to the mirror and from the mirror to your foot. The horizontal dotted lines show the heights of the mirror's top and bottom. The distance from the top of your hat to the dotted line indicating the top of the mirror is the same as the distance from this line to your eye. The distance from your eye to the dotted line indicating the bottom of the mirror is the same as that from this line to your foot. Thus, you can see twice the length of the mirror.



Try drawing the figure closer to the mirror or farther away. Although the angles change, the part of the body seen in the mirror is always twice the length of the mirror.

Try This:

- Use masking tape to tape one end of each of three long strings to the bottom of the mirror.

  Back up from the mirror and hold one string up to your eye. Have your friend hold a second string horizontal to the floor and tape it to your body where it hits. Standing straight up, look in the bottom of the mirror and have your friend tape the third string to the lowest point on yourself that you can see. (Leaning forward can affect the outcome so try to stand straight up.)

  Have your friend carefully measure the distance from your eye to the second string and from the second string to the third string. Are the measurements close to being the same? (They should be!)

  Back up and try it again. Did these measurements change?

Additional Challenges:

1. If a person can see his entire body in a 42 in. mirror, how tall is he?
2. If you had a 42-inch mirror, where would you place it on the wall to see your whole body?
3. If you want to be able to see yourself head-to-toe in a mirror, how big does the mirror have to be and where on the wall should you hang it?

Things to think about:

- Many fun houses and shopping malls have curved mirrors. What do you see when you look in such a mirror? What happens when you step back from this mirror?
- What type of mirrors make you look larger? smaller?
- Suppose a friend is directly behind you, standing still, when you look in a mirror. When you move back and forth, do you see more or less of your friend?
- If you stand in front of a counter looking at a mirror and back up, you see more of yourself. Why?

Did You Know That?

- According to the BBC World News, the Russian space project, Znamya, has been trying to place a large mirror in space in order to reflect the sunlight down on northern cities.
- The tallest human on record was 8 ft 11.1 in. tall. Robert Pershing Wadlow (1918–1940) would have needed a 54 inch mirror to see himself from head to toe.
- A reflection is an isometry, a geometric motion that preserves distance.

Resources:

Books:

- Desoe, Carol. *Activities for Reflect-It™ Hinged Mirror*. White Plains, NY: Cuisenaire Company of America, 1994.
- Walter, Marion. *The Mirror Puzzle Book*. Jersey City, NJ: Parkwest Publications, Inc., 1985.

Film:

- Harvard-Smithsonian Center for Astrophysics. *A Private Universe*. Washington, DC: Annenberg/CPB Math and Science Collection, 1987. [Materials include a 20-minute videocassette and *A Private Universe Teacher's Guide*.] **www.learner.org**

Website:

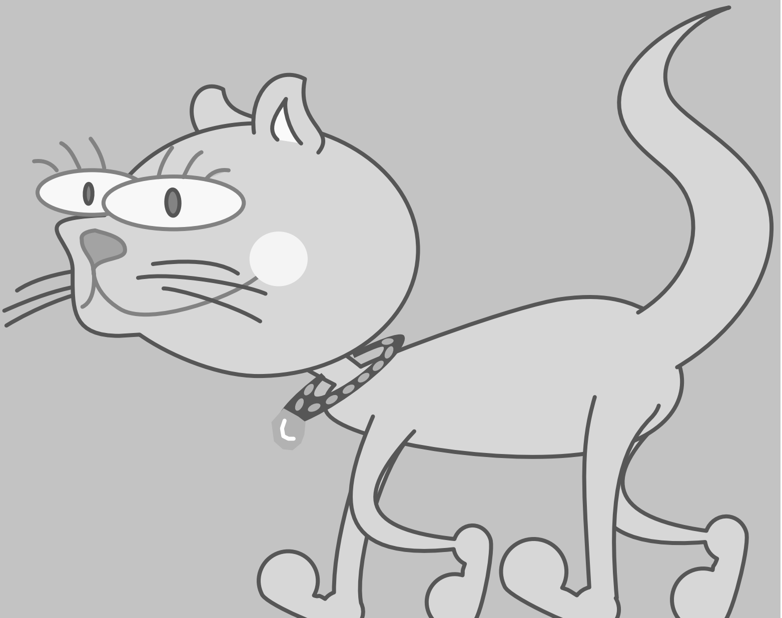
- Link to article on the Russian space mirror project, Znamya 2.5: **news.bbc.co.uk/hi/english/sci/tech/newsid_272000/272103.stm**

Answers to Additional Challenges:

(1.)
What you see in the mirror is twice as long as the mirror. A person 84 inches tall or less can see all of himself in such a mirror.

(2.)
This depends on your height. The top of the mirror should be at least 2 to 3 inches above your eye level so you can see the top of your head. Unless you are over 84 inches tall, you will then be able to see your feet.

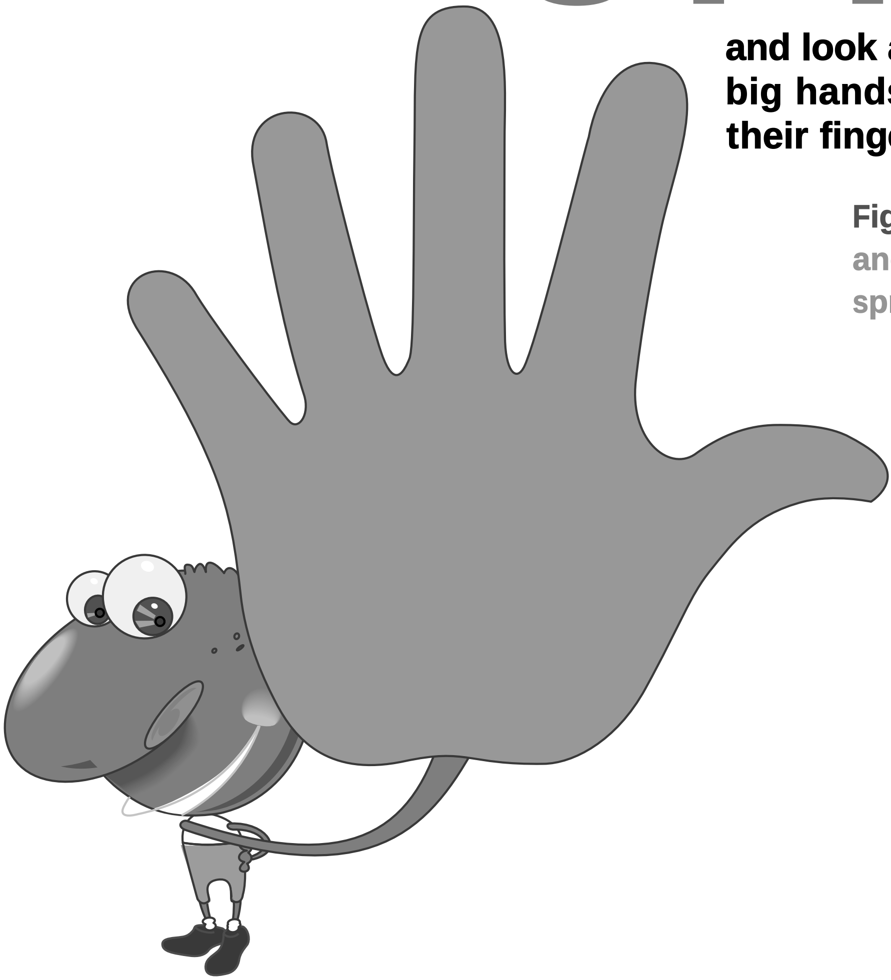
(3.)
(See the first Additional Challenge.) The length of the mirror depends on your height. You should hang it so that the bottom of the mirror is halfway between your feet and eyes.



Math Challenge #10
WHAT'S MY ANGLE?

FigureThis!
Math Challenges for Families

SPREAD out your fingers



and look at your hand. Do people with big hands have larger angles between their fingers?

Figure This! Estimate the measures of the angles between your fingers when you spread out your hand.

Hint: When you hold your hand so that your thumb and index finger form an "L," the angle formed measures about 90°.

Angles are important geometric shapes. They are used in designing many things from airplanes to golf clubs.

Answer:
The size of your hand does not make any difference in the size of the angles.
The measures are approximately 90°, 45°, 20°, and 20°.

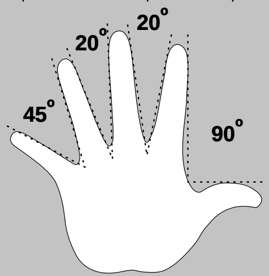
Figure This!

Get Started:

The angle measure between your thumb and index finger when your hand is shaped like an “L” is about 90˚. How do the angle measures between your other fingers compare to this angle measure? Is the angle measure larger or smaller? Half as big? Draw the 90˚ angle between your thumb and index finger. Sketch an angle with half that measure. Use your sketch to estimate the measures of the other angles.

Complete Solution:

The largest angle, between the thumb and index finger, measures about 90˚. (A 90˚ angle is a right angle.) The other angle measures vary somewhat depending on the person. One possibility is shown below:

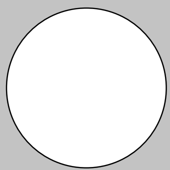


The lengths of the fingers do not affect the angle measures.

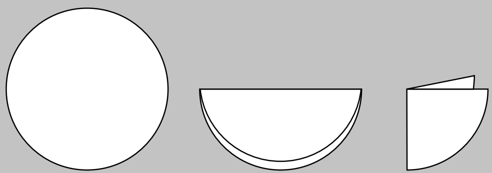
Try This:

- Measure an angle by making your own angle measurer or protractor using the following steps.

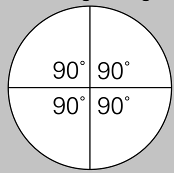
1. Cut out a circular piece of waxed or other lightweight paper. (You may want to trace the bottom of a large cup.)



2. Fold the circle in half, and then fold the result in half.



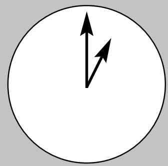
3. Unfold the paper and open the circle. You should see four right angles at the center of the circle. Each right angle has measure 90˚.



4. Refold the circle on the same crease marks. Then fold the paper in half one more time. Unfold the paper to reveal eight angles, each of which measures 45˚.
5. Refold the paper on the previous creases. Then fold in half one more time. How large is each new angle? Label each new angle with the appropriate measure.
6. To measure an angle with your protractor, place the center of the circle on the point of the angle and line up one of the creases with a side of the angle. Estimate the angle’s measure by finding where the other side aligns on the protractor.

Additional Challenges:

1. Suppose a bicycle wheel turns around exactly once. This is a 360˚ revolution. How far would the bicycle wheel have moved on the ground?
2. On a compass, north has a heading of 0˚. What is your heading when you are going east, south, or west?
3. What is the measure of the angle between the hands on a clock if one hand is on the number 12 and the other is on the number 1?



4. What do you think a negative angle might be?

Things to Think About:

- Why do you think Tiger Woods worries about the angle at which he hits a golf ball?
- What angles are important in designing a bicycle? Why?
- Many chairs recline or lean back. What kind of angles can you make in a reclining chair or in the driver’s seat of a car?
- Angles are formed by tree branches and in the veins of a leaf. How big are these angles?
- What types of angles can you find on a cereal box?

Did You Know That?

- The legs and upper body of an astronaut floating in space form a natural angle of about 128°.
- People recovering from knee surgery track their progress by achieving flexibility to a certain angle measure.
- To measure angles between joints, orthopedic surgeons and physical therapists, use an instrument called a goniometer.
- Angle measure is related to the number system of the ancient Babylonians.
- Angles can be measured in radians and grads as well as degrees.

Resources:
Books:

- Desoe, Carol. *Activities for Reflect-It ™ Hinged Mirror.* White Plains, NY: Cuisenaire Company of America, 1994. **www.cuisenaire-dsp.com**
- Greenes, Carole, Linda Schulman-Dacey, and Rika Spungin. *Geometry and Measurement*. White Plains, NY: Dale Seymour Publications, 1999. **www.cuisenaire-dsp.com**
- Page, David A., Philip Wagreich, and Kathryn Chval. *Maneuvers with Angles*. White Plains, NY: Dale Seymour Publications 1993. **www.cuisenaire-dsp.com**

Answers to Additional Challenges:

(1.) Answer:
The answer depends on the size of the wheel. You can find this distance by conducting an experiment. Mark the tire at a point where it touches the ground and mark the point on the ground. Roll the bicycle ahead until the mark is once more on the ground and mark the distance it traveled. Measure the distance between the two marks on the ground. Another way to find the answer is to measure the diameter of the wheel, then use the fact that the distance around a circle is its circumference. Circumference of a circle = $\pi \bullet$ diameter where π is about 3.14.

(2.) Answer:
90°, 180°, and 270°, respectively.

(3.) Answer:
The angle has measure 30° because 12 angles of 30° each make 360°. Each of the other similarly formed angles also measure 30°.

(4.) Answer:
If positive angles are measured in one direction (for example, counterclockwise), negative angles would be measured in the opposite direction.

Math Challenge #11

PLAY BALL!

Figure This!
Math Challenges for Families

What's round, hard, and sold for $3 million?

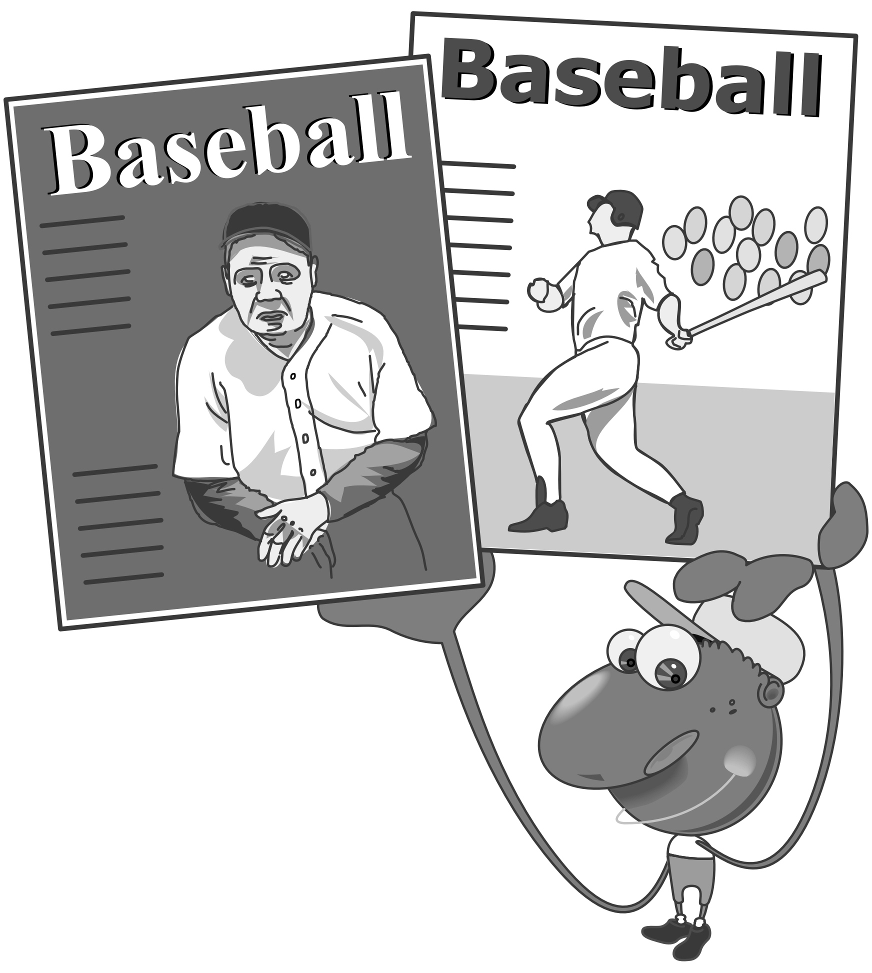


Figure This! Mark McGwire became baseball's home run king in 1998 with 70 home runs. His 70th home run ball sold for slightly over $3 million in 1999. Babe Ruth, an earlier home-run king, hit 60 in 1927. His home-run ball was donated to the Hall of Fame. Suppose that Ruth's ball was valued at $3000 in 1927 and, like many good investments, doubled its value every seven years. Would you rather have the value of Ruth's ball or McGwire's?

Hint: How many times would you need to double the value of Ruth's ball to reach the value of McGwire's?

Compound interest and rate of change over time affect many quantities. Bankers, stockbrokers, and population biologists have to understand this kind of change in their work.

Answer:
You decide. If the value of Ruth's ball started at $3,000 doubled every seven years since 1927, its value in 1997 would be approximately $3,072,000.

Figure This!

Get Started:

Assume that Ruth's ball was valued at $3000 in 1927. What was its value seven years later? Try making a table.

Complete Solution:

Suppose Ruth's ball had a value of $3000 in 1927. If the price doubled in seven years, the ball would be worth $6000 in 1934. In seven more years, its value would double again.

Year	Value
1927	3000
1934	$2 \cdot 3000 = 6000$
1941	$2 \cdot 2 \cdot 3000 = 2^2 \cdot 3000 = 12{,}000$
1948	$2 \cdot 2 \cdot 2 \cdot 3000 = 2^3 \cdot 3000 = 24{,}000$
1955	$2 \cdot 2 \cdot 2 \cdot 2 \cdot 3000 = 2^4 \cdot 3000 = 48{,}000$
	. . .
1997	$2^{10} \cdot 3000 = 3{,}072{,}000$

The year 1997 was 70 years after 1927, so there would be 10 sets of 7 years during that time. By 1997, Ruth's ball would have a value of $3,072,000. Since it would have a greater value than McGwire's in 1997, it would have a greater value in 1999.

Try This:

- Find out how much annual interest you could earn on a savings account at a local bank. If you invest money at this rate, how long would it take your money to double? Are there any conditions you would have to consider?
- As a family, talk about any loan that you may have for a college education, a house, a car, or an appliance. What was the original price? What is the total amount that you would spend by the time the loan is paid off completely?

Additional Challenges:

1. If you invest $10 at an annual interest rate of 7%, in about how many years will your money double?
2. If your investment earns 5% annual interest, how many years will it take to double your money?
3. What annual interest would you have to earn to double your money in seven years?
4. A new baseball costs about $6. How many baseballs could you buy with $3 million?
5. Suppose the number of water lilies in a pond doubles every day. If the pond was half covered on Monday, when was it only one-fourth covered with lilies? When would it be completely covered?
6. What would be the value of Babe Ruth's ball in 1999?

Things to Think About:

- A banker's general rule to find the number of years needed to double an investment is to divide 70 by the interest rate. Try this rule on the challenges.
- Banks have computers programmed to use fractional powers to calculate interest earned on savings accounts and interest owed on loans.

Did You Know That?

- The $3,000,000 price of Mark McGwire's baseball was 23 times that of any baseball previously sold and five to six times the highest price paid for any other sports artifact.
- A number, *e*, named for Leonhard Euler (1707-1783), is used in computing continuous interest.

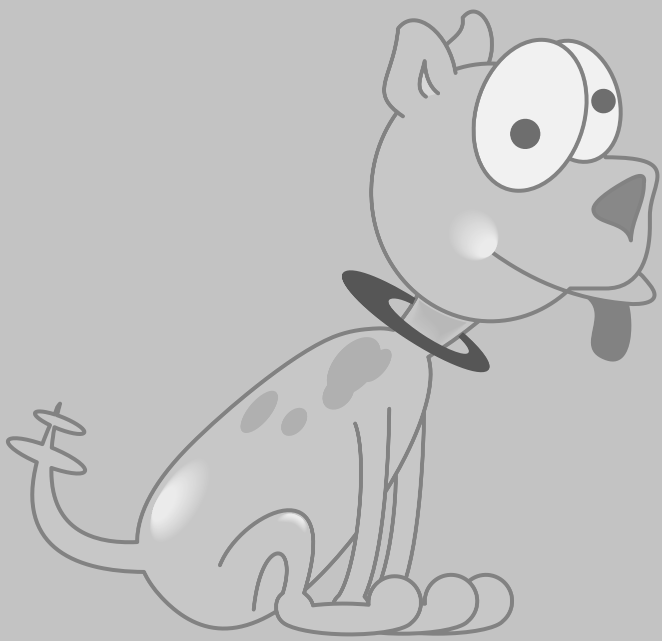
Resources:

Books:

- Sports Illustrated. *Home Run Heroes—Mark McGwire, Sammy Sosa and a Season for the Ages*. New York: Simon & Schuster, 1998.
- "Baseball." *The Encyclopedia Americana*. International Edition. Bethel, CT: Grollier, Inc. 1998.
- "Mark McGwire vs. Sammy Sosa: The 1998 Home Run Race." *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.majorleaguebaseball.com**
- **www.baseballhalloffame.org**



Answers to Additional Challenges:

(1.)

Assuming that no withdrawals are made and that all interest is reinvested, the balance will reach $20 during the tenth year.

(2.)

The money will double during the 14th year.

(3.)

About 10.4%.

(4.)

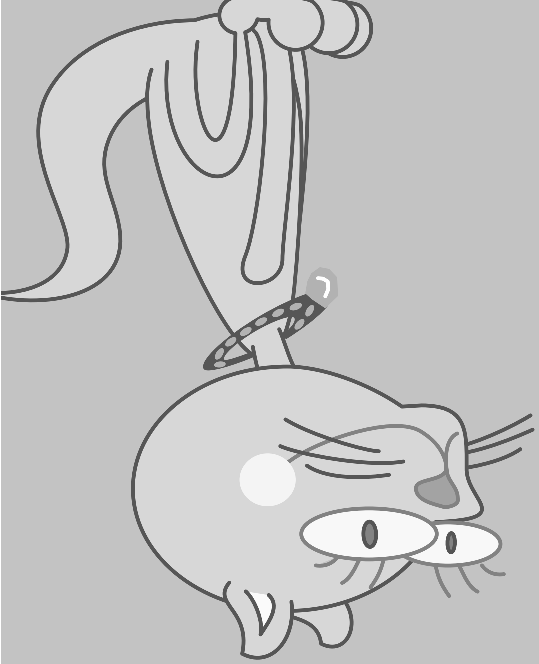
500,000 balls.

(5.)

The pond would be one-quarter covered on Sunday. By Tuesday, it would be totally covered.

(6.)

The year 1997 was 70 years after 1927, so there are 10 sets of 7 years during that time. The value would double 10 times, giving $2^{10} \cdot 3000$, or $3,072,000. The year 1999 is 72 years after 1927, so there are 72/7 sets of 7 years. The value would be $2^{72/7} \cdot 3000$. Using a calculator, this is about $3,744,810.



Math Challenge #12 CHOCOLATE

Figure This! Math Challenges for Families

Which of these chocolate-covered cookies would YOU eat?

mmm... chocolate

Figure This! Suppose you love chocolate. The top of each cookie is covered with the same thickness of chocolate. If you wanted to choose the cookie with more chocolate, which one would you pick?

Hint : Think about how to measure the area of the top of each cookie.

There are no simple ways to find the exact areas of irregular shapes, such as land masses or living cells. Estimating these areas can be important in land-use planning and medical research.

Answer:

The chocolate lover would choose the cookie with the rough edges.

Figure This!

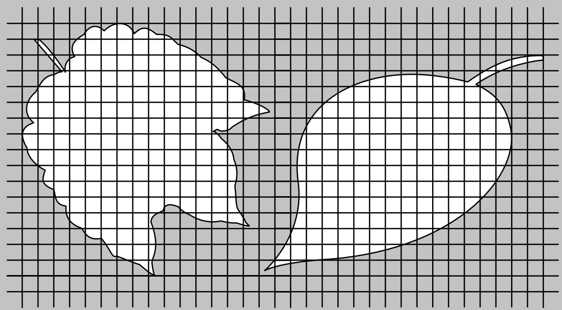
Get Started:

Trace the cookies on a sheet of paper and cut them out. How do you think their areas compare? Would graph paper help?

Complete Solution:

There are many ways to do this challenge.

- Trace the cookies on graph paper and count the number of squares each one covers. The smaller the squares, the better the estimate of the area.



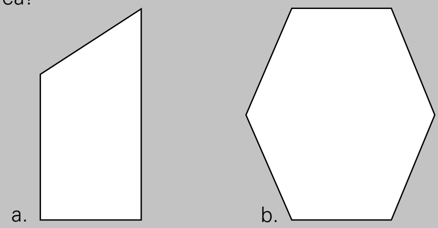
- Cut out the cookies, put one on top of the other and cut off the parts of one that are not covered by the other. Try to fill in the extra space with the parts you cut off. If you cannot cover all of the first cookie with the parts of the second, the first one is larger. If you have pieces of the second cookie left when the first is covered, the second one is larger.
- Cover each cookie with something small (cereal or rice) and then compare the two quantities.

Try This:

- Find the area of a nearby baseball field.
- Find an irregular shape in the room around you. Estimate its area.
- Draw an irregular shape on a piece of paper. Choose some point inside the shape and call that the "center." Find the lengths from the center to different points on the edge of the shape. [If a segment goes outside the shape, add the lengths of the pieces that are inside the shape. Find the average of all the lengths. Let this average be the "radius" of the shape. Use the formula for the areas of a circle ($\pi \times$ radius $\times$ radius, or about $3.14 \cdot r \cdot r$). This should be a good estimate of the area.

Additional Challenges:

1. You can use a string to find the perimeter of each cookie. Some people might think that the cookie with the greater perimeter will have the greater area. Do you agree? Why or why not?
2. You can find the area of some figures by dividing them into rectangles, squares, or triangles. How could you divide the shapes below to find the area?



Things to Think About:

- Some people say that a coastline has infinite length. What could they mean by this?
- When people talk about buying so many yards of carpet, they are really talking about square yards; with yards of concrete or sand, they are really talking about cubic yards.

Did You Know That?

- Square measure is reasonable to find areas because square regions can cover a flat surface with no overlapping and no holes.
- Although you usually read only about the area covered by an oil slick, it also has volume.
- A planimeter is a tool that measures the area of irregular shapes by tracing the perimeter of the figure. A planimeter involves the concepts of polar coordinates.

Resources:

Books:

- Gravemeijer, K., M. A. Pligge, and B. Clarke. "Reallotment." In *Mathematics in Context*. National Center for Research and Mathematical Sciences Education and Freudenthal Institute (eds.). Chicago: Encyclopaedia Britannica Educational Corporation, 1998.
- Lappan, G., J. Fey, W. Fitzgerald, S. Friel, and R. Phillips. *Connected Mathematics: Covering and Surrounding*. Palo Alto, CA: Dale Seymour Publications, 1996.

Answers to Additional Challenges:

(1.)

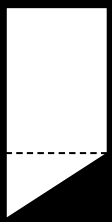
For the same perimeter, some shapes have smaller areas, while others have greater areas.

(2.)

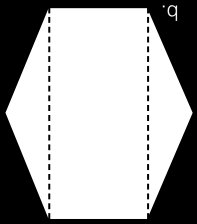
a. Divide the shape into a triangle and a rectangle.

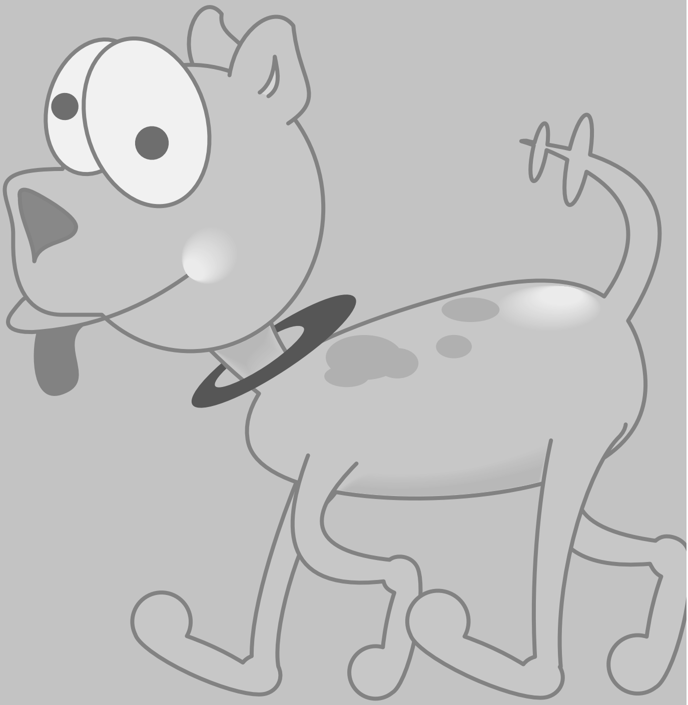
b. Divide the shape into a rectangle and two triangles.

a.



b.





Math Challenge #13
THIRTEEN! OH NO!

Figure This!
Math Challenges for Families

Are you SUPERSTITIOUS?
Do you avoid the number 13?

Figure This! Is there a Friday the 13th every year?

Hint: If January 1 were on Monday, on what day of the week would January 13 fall? What about February 1 and February 13? Other months?

Reasoning about patterns with numbers and dates helps to develop logical thinking. Matching such patterns determines the annual dates of some national holidays as well as being an operating principle for some machines.

Answer:
There is at least one Friday the 13th every year.

Figure This!

Get Started:

- Make a list. If January 1 is on a Monday, on what day is January 13? On what day is the first day of the next month? The 13th of the next month? Suppose January 1 is on a Tuesday?
- Get a calendar and check to see how the dates fall. How many different calendars are possible? (Remember leap years.)

Complete Solution:

There are 14 different calendars. (Check the website for all calendars.) There are seven possible calendars for non-leap years: one with January 1 on each day of the week. Leap years give seven more calendars, for a total of 14. For example, if January 1 is on a Wednesday, you have the following list:

January 1	Wednesday	April 1	Tuesday
January 13	Monday	April 13	Sunday
February 1	Saturday	May 1	Thursday
February 13	Thursday	May 13	Tuesday
March 1	Saturday	June 1	Sunday
March 13	Thursday	June 13	Friday

Because June 13 is a Friday, you can stop.The chart below shows the total number of Friday the13ths for all 14 calendars.

When January 1st is on	Non-leap Year months in which Friday the 13th will occur	Leap Year months in which Friday the 13th will occur
Monday	April, July	September, December
Tuesday	September, December	June
Wednesday	June	March, November
Thursday	February, March, November	February, August
Friday	August	May
Saturday	May	October
Sunday	January, October	January, April, July

Try This:

- Use an almanac or an encyclopedia to discover how the months were named and why they have different numbers of days.
- If you had been born on February 29, how many birthdays would you have had by this year?
- Thirteen may be a lucky number for the USA. Think of at least one reason why.
- Study a dollar bill to see if you can find "13" objects in common.
- Use an almanac or an encyclopedia to research why the number 13 is considered by some to be unlucky.

Additional Challenges:

1. Why is the year 2000 a leap year when 1900 was not?
2. If your VCR cannot handle the year 2000, to what year can you set it back so that the days will still be the same?

Did You Know That?

- The word triskaidekaphobia means fear of the number 13.
- Some hotels do not have a floor numbered 13 because of people's fears of 13.
- The calendar is based on the movements of the sun and the moon.
- There have been many different calendars in the past. The current calendar began in 1582 when Pope Gregory XIII decreed that the day following October 4 should be October 15 to catch up for the days lost using the previous calendar.
- The calendar correction did not take place in Great Britain and its colonies (including those in North America) until 1752.
- The Chinese New Year falls anywhere from late January to the middle of February. The Chinese Lunar Calendar is based on cycles of the moon, with a complete cycle requiring 60 years (5 cycles of 12 years each).
- The mathematics of modular arithmetic is used to find answers to challenges like this one.

Things to think about:

- Why do you think that we have seven days per week with 52 weeks per year?
- Hotels that rename the 13th floor as the 14th floor still have a 13th floor.
- The words September, October, and November are derived from Latin words *septem*, *octo*, and *novem* meaning seven, eight and nine respectively, but the months are not the seventh, eighth, and ninth months. Why?

Resources:

Book:

- *The World Almanac and Book of Facts 1999.* Mahwah, NJ: World Almanac Books, 1999.

Websites:

- **www.julian12.com/history.htm**
- **www.stjohndc.org/what/9609ca1.htm**

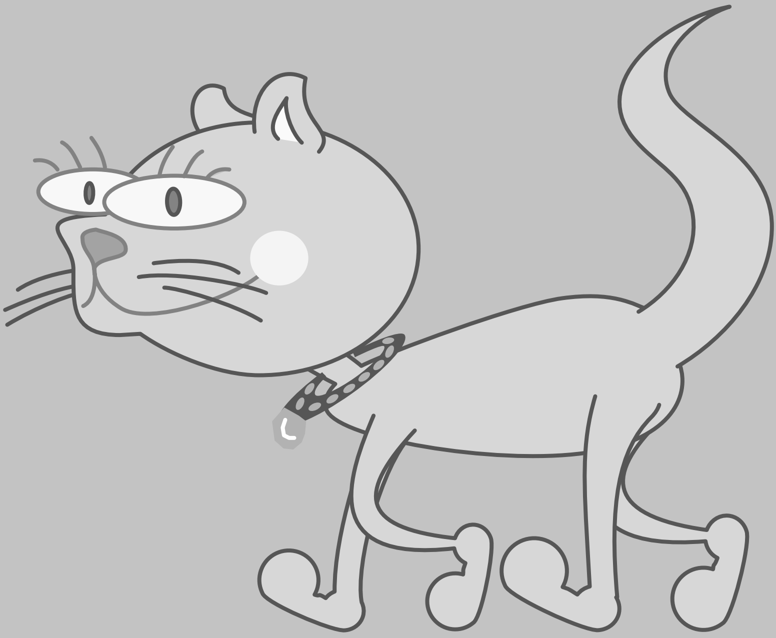
Answers to Additional Challenges:

(1.) Answer:

Leap years typically occur every four years, except at the turn of a century. Only those centuries divisible by 400 are leap years.

(2.) Answer:

1972



Math Challenge #14 BATTLING AVERAGES

FigureThis! Math Challenges for Families

Who's on first today?

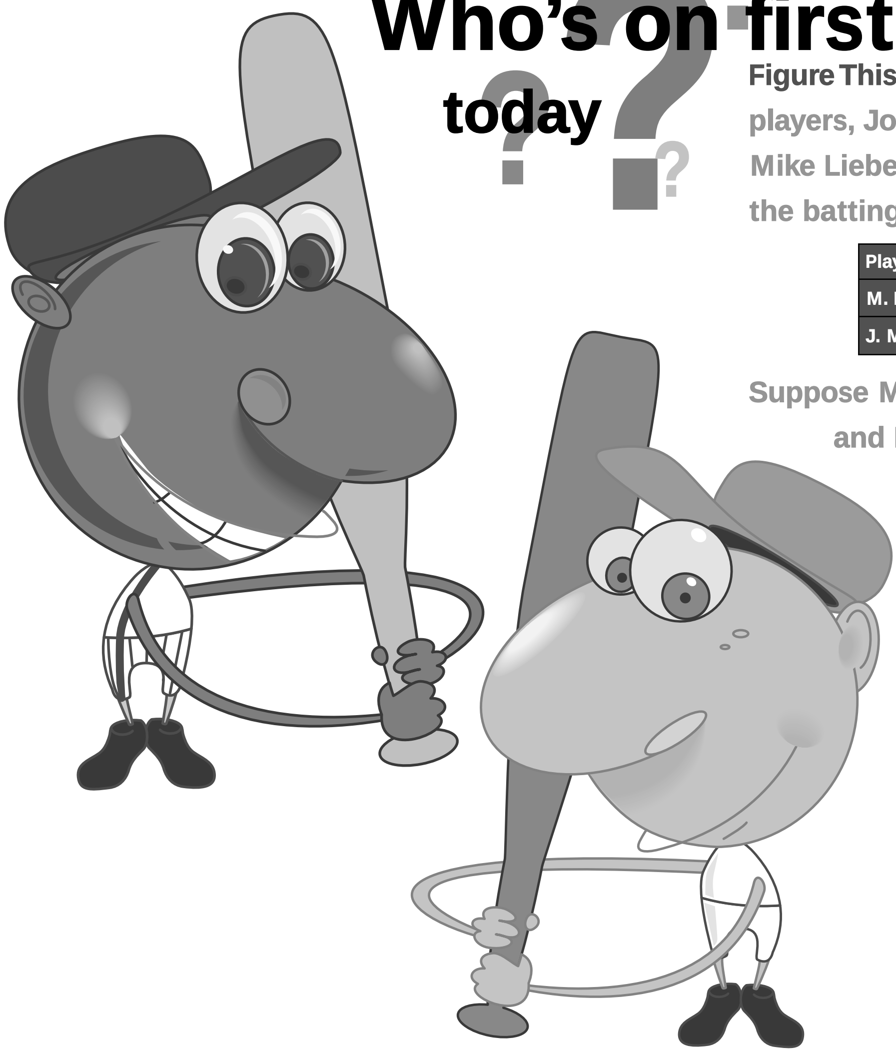


Figure This! In May 1999, two National League baseball players, Joe McEwing of the St. Louis Cardinals and Mike Lieberthal of the Philadelphia Phillies, each had the batting averages shown below.

Player	Team	At Bats	Hits	Batting Average
M. Lieberthal	Phillies	132	45	.341
J. McEwing	Cardinals	132	45	.341

Suppose McEwing then batted .800 (4 hits in 5 at bats), and Lieberthal was perfect (3 hits in 3 at bats). Which player now has the higher batting average? Are you surprised?

Hint: Batting average = $\frac{\text{Number of hits}}{\text{Number of at bats}}$ (rounded to the nearest thousandth)

An average is a tool for helping us understand and compare sets of numbers. Sports, medicine, and insurance are three of the many fields that use averages.

Answer:
McEwing has the higher average.

Figure This!

Get Started:

Make a new table using the updated information.

Complete Solution:

Both players had 45 hits in 132 at bats. Then with the statistics from the next at bats, McEwing's average is 49/137 or about .358 while Lieberthal's batting average is 48/135 or about .356. [≈ is a symbol that indicates "approximately equal to."]

Player	Team	At Bats	Hits	Batting Average	Next At Bats	Next Hits	New Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$	3	3	$\frac{48}{135} \approx .356$
J. McEwing	Cardinals	132	45	$\frac{45}{132} \approx .341$	5	4	$\frac{49}{137} \approx .358$

McEwing has the higher batting average. One way to make sense of this unexpected result is to imagine that McEwing gets 3 hits in his first 3 at bats while Lieberthal also gets 3 hits in 3 at bats. Then the pair is still tied. During McEwing's last 2 at bats, he gets 1 hit. This average of 1 for 2, or .500, is better than his current average, so his batting average goes up.

Try This:

- Check out some other sports. What statistics are collected? In which ones are averages computed?

Additional Challenges:

1. Suppose New York Yankee Chili Davis and Lieberthal have the batting averages shown.

Player	Team	At Bats	Hits	Batting Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$
C. Davis	Yankees	137	47	$\frac{47}{137} \approx .343$

If Davis bats 3 for 3 while Lieberthal has 4 hits in his next 5 at bats, who now has the higher average?

2. What number can be added to both the numerator and the denominator of a fraction so that the new fraction is equal to the original fraction?

Did You Know That?

- Rogers Hornsby of the St. Louis Cardinals had the highest season batting average in modern baseball history. In 1942, Hornsby hit a remarkable .424.
- The word fraction comes from the Latin word ***frangere*** meaning "to break."
- The ancient Egyptians primarily used fractions whose numerators were 1.
- Batting averages are typically spoken as whole numbers but are actually decimals.

Things to Think About:

- If you add all the numerators and all the denominators in a set of equal fractions, the result is a fraction equal to those in the original set.
- If you got an A on a test and a C on homework, do you have a B average?
- In the challenge, two players began the day with the same batting average. Lieberthal batted 1.000 while McEwing batted .800, yet McEwing ended the day with the higher cumulative batting average. Such unexpected results are called Simpson's Paradoxes, after Thomas Simpson, a mathematician who worked in the 1700s.
- Lou Abbott and Bud Costello had a comedy routine about baseball called "Who's on First?"

Resources:

Book:

- *The World Almanac and Book of Facts 1999*. Mahwah, NJ: World Almanac Books, 1999.

Websites

- Baseball Hall of Fame:
  www.baseballhalloffame.org
- Exploring Data:
  curriculum.qed.qld.gov.au/kla/eda/sim_par.htm
- UCLA: Simpson's Paradox:
  www.stat.ucla.edu/~abraverm/Simpson/simpson.html
- Simpson's Paradox Plaything:
  www.stat.ucla.edu/~abraverm/Simpson/simpsonpt.html
- **www.aentv.com/home/golden/colgate.htm**

Answers to Additional Challenges:

(1.)
Lieberthal's average is better, although not by much.

Player	Team	At Bats	Hits	Batting Average	Next At Bats	Next Hits	New Average
M. Lieberthal	Phillies	132	45	$\frac{45}{132} \approx .341$	5	4	$\frac{49}{137} \approx .358$
C. Davis	Yankees	137	47	$\frac{47}{137} \approx .343$	3	3	$\frac{50}{140} \approx .357$

(2.)
The number 0.

Math Challenge #15

BIG TREES

Figure This!
Math Challenges for Families

Have you ever seen a tree **big** enough to drive a car through?

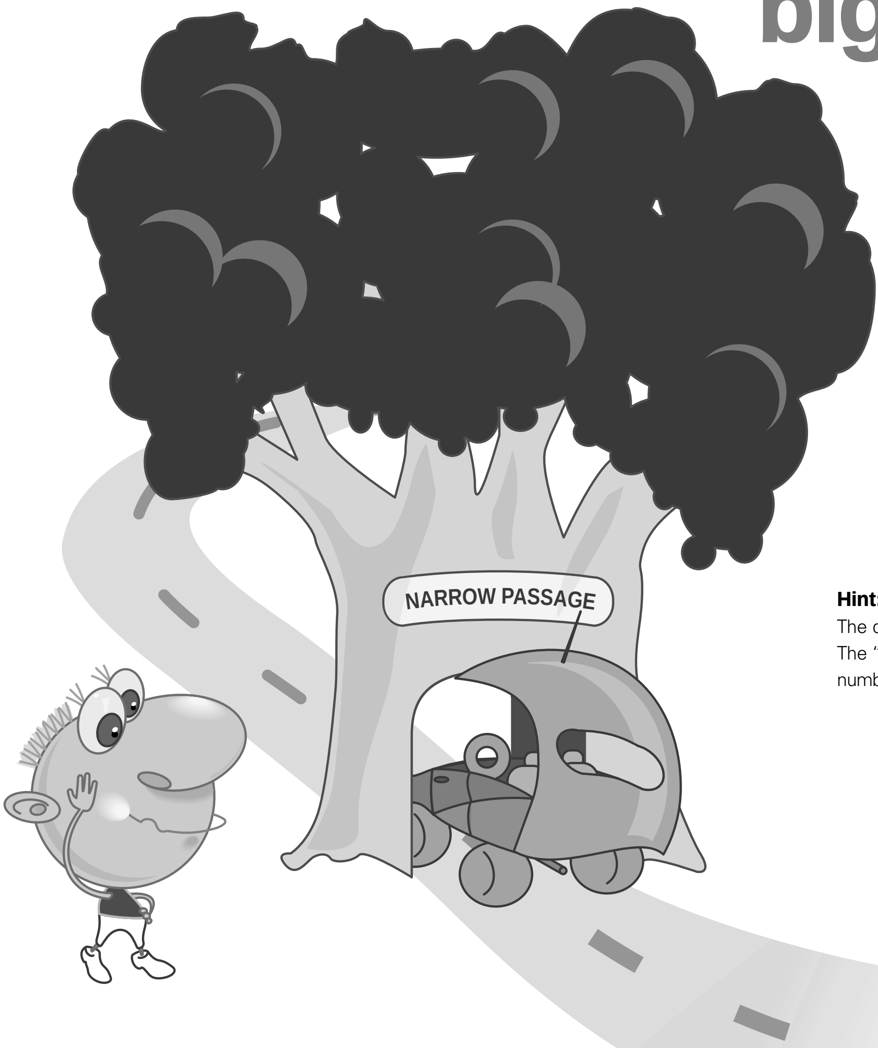


Figure This! Are any of the "National Champion" trees in the table below wide enough for a car to drive through?

Tree	Girth 4.5 ft above ground In inches	Height in Feet	Location
American Beech	279	115	Harwood, MD
Black Willow	400	76	Grand Traverse Co., MI
Coast Douglas Fir	438	329	Coos County, OR
Coast Redwood	867	313	Prairie Creek Redwoods State Park, CA
Giant Sequoia	998	275	Sequoia National Park, CA
Loblolly Pine	188	148	Warren, AR
Pinyon Pine	213	69	Cuba, NM
Sugar Maple	274	65	Kitzmiller, MD
Sugar Pine	442	232	Dorrington, CA
White Oak	382	96	Wye Mills State Park, MD

Hint:

The distance around a tree is its girth. The distance around a circle is its circumference. The "width" of a circle is its diameter. Finding the circumference of a circle involves the number π, about 3.14. The circumference of a circle is π times the diameter.

Measurement is important in many jobs. Carpenters, biologists, foresters, designers, and publishers use measurement formulas in their work.

Answer:

Only the coast redwood and the giant sequoia are clearly big enough. If two-feet "walls" are enough to hold up the tree around a six-foot wide car, then the black willow, the coast Douglas fir, the sugar pine, and the white oak are big enough as well.

Figure This!

Get Started:

What do you need to know about a car before you can answer the question? about the tree? What do you know about circles? How is the distance around a tree related to the width of a tree?

Complete Solution:

To find the diameter of a circle when you know the circumference, you divide the circumference by π (about 3.14). For example, the black willow has a girth of 400 inches and since $400 \div 3.14$ is about 127, the black willow is about 127 inches wide, enough for a car to drive through and still leave at least 2 ft on each side.

Tree	Girth 4.5 ft above ground In inches*	Diameter (Girth/π) In inches*	Height In Feet
American Beech	279	89	115
Black Willow	400	127	76
Coast Douglas Fir	438	139	329
Coast Redwood	867	276	313
Giant Sequoia	998	318	275
Loblolly Pine	188	60	148
Pinyon Pine	213	68	69
Sugar Maple	274	87	65
Sugar Pine	442	141	232
White Oak	382	122	96

*Measurements rounded to nearest whole number.

Try This:

Find a tree in your yard or a park. Estimate the diameter of the tree.

Additional Challenges:

1. Are any of the National Champion trees taller than a 15-story building?
2. How many people holding hands would it take to go around the giant sequoia?

Things to Think About:

- How do you think foresters estimate the weight of a living tree?
- How do foresters estimate the number of feet of lumber in a tree?
- Why is the girth measured 4.5 feet above the ground?

Did You Know That?

- There are approximately 825 native and naturalized species of trees in the United States.
- The oldest living tree is believed to be a California bristlecone pine tree named Methuselah, estimated to be 4700 years old.
- The world's largest known living tree, the General Sherman giant sequoia in California, weighs as much as 41 blue whales or 740 elephants, about 6167 tons.
- In about 1638, Galileo Galilei (1564-1643) suggested that trees could grow only to be about 300 feet tall because of factors involving form and material.

Resources:

Book:

- *The World Almanac and Book of Facts 1999*. Mahwah NJ: World Almanac Books, 1998.

Website:

- National Register of Big Trees, American Forests **www.amfor.org**

Answers to Additional Challenges:

(1.)

Assuming that a typical story is 12 feet high, 15 stories would be about 180 feet high. Thus, the coast Douglas fir, the coast redwood, the giant sequoia, and the sugar pine would be taller than a 15-story building. The loblolly pine would be about the same height.

(2.)

Assuming that the arm span (with arms outstretched when holding hands) of a typical person is about 60 inches, it would take about 17 people to encircle the tree.

Math Challenge #16 BEANIE BABIES

FigureThis! Math Challenges for Families

PROFIT - or - LOSS?

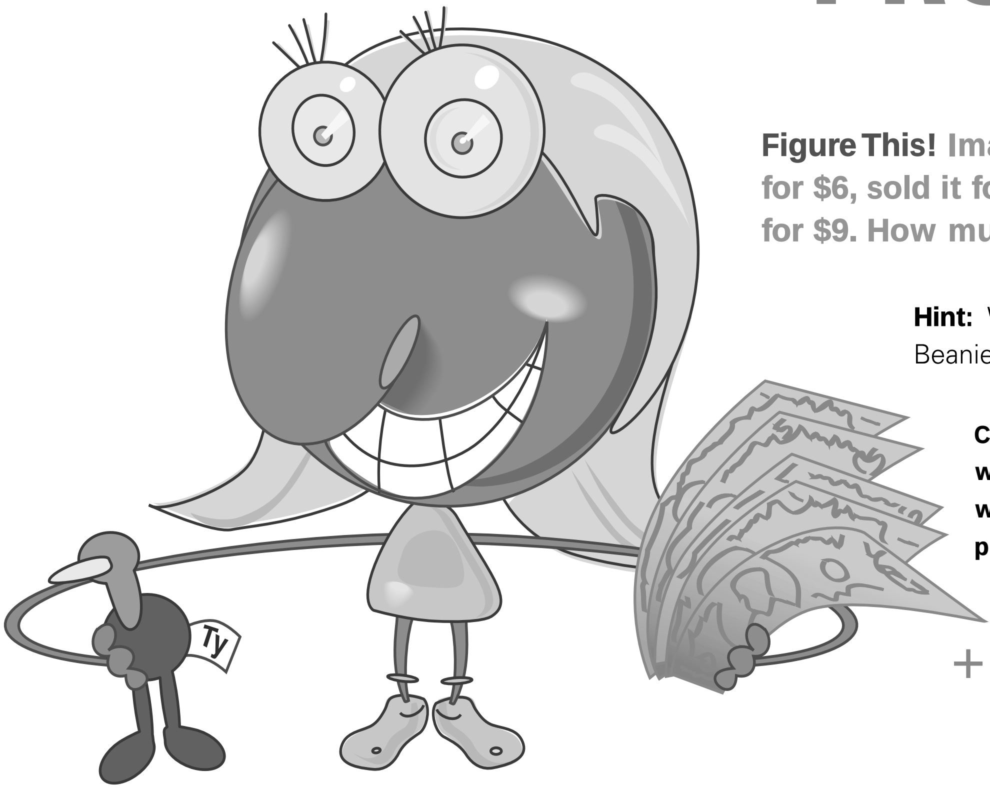


Figure This! **Imagine that you bought a Beanie Baby™ for \$6, sold it for \$7, bought it back for \$8, then sold it for \$9. How much profit did you make?**

Hint: Would it change your calculations if the second Beanie Baby™ were different than the first?

Calculations can often be done in several different ways. As long as the reasoning is correct, the result will be the same. Calculating, predicting, and reporting profit and loss are critical business skills.

\+ ? - ? + ? - ? + ? - ? +

Answer:

You made \$2.

Figure This!

Get Started:

Think of this challenge as two separate problems. How much money did you make on the first sale? on the second?

Complete Solution:

In this situation, profit equals the selling price minus your cost. On the first sale, you made a profit of $1. On the second, you made another $1. The total profit was $2.

Paid	Sold	Profit or Loss
–$6	+$7	+$1
–$8	+$9	+$1
	Total	+$2

Try This:

Find the stock-market listings on television, in the newspaper, or on the Internet. Choose a company or two and pretend to buy their stock. Follow the listing for a week. Would you have gained or lost money on your investment if you did not have to pay someone to buy and sell your stocks?

Additional Challenges:

1. Your class is sponsoring a dance. The expenses include $450 for a DJ, $125 for refreshments, $45 for decorations, and $30 for advertisements. Judging from previous dances, you expect to take in at least $150 on the refreshment stand alone. If you charge $5 per person to attend the dance, how many people must attend so your class can make a profit?

2. Suppose you bought 200 shares of stock at $45 a share. When the price per share went up $5, you sold 100 shares. Several weeks later, the price per share was down $10 from your previous selling price, so you sold the remaining 100 shares. How much money did you make if you don't have to pay someone for buying and selling the stock?

Things to Think About:

- How can people make money by repeatedly buying and selling the same item?
- Can a store always make a larger profit by charging more?
- What factors does a business consider before setting prices?

Did You Know That?

- The Securities and Exchange Commission (SEC), as a part of its job, monitors profits and losses of businesses that are publicly traded.
- The Beanie Baby™ is an invention of H. Ty Warner of Ty, Inc. He wanted to market an inexpensive toy that children could buy themselves. The original nine Beanies—a moose, bear, dolphin, frog, platypus, lobster, whale, dog, and pig—were first made available to the public in 1994, primarily in the Chicago area. According to the 1999 edition of *Toys and Prices*, 1999, a dark blue Peanut the Elephant in mint condition was valued at $4200.
- From October 1, 1981, to September 30, 1982, the American Telephone and Telegraph Co. (AT&T) made a net profit of $7.6 billion.
- As of 1997, the most active common stock on the New York Stock Exchange was Compaq Computer Corporation, with a volume of 1231.6 million shares either bought or sold during a year.

Resources:

Books:

- Brecka, Shawn. *The Beanie Family Album and Collectors Guide*. Norfolk, VA: Antique Trader Books, 1998.
- Korbeck, Sharon. *Toys and Prices*. 6th Edition. Iola, WI: Krause Publications, 2000.
- *The Guinness Book of World Records*. New York: Bantam Books, 1998.
- *The World Almanac and Book of Facts*, 1999. Mahwah, NJ: World Almanac Books, 1998.
- Gardner, Martin (ed.). *Mathematical Puzzles of Sam Loyd.* New York: Dover Publications, Inc. 1959.

Websites:

- **www.ty.com**
- **www.historychannel.com** (search on Toys & Games)

Answers to Additional Challenges:

(1.)
more than 100 people.

(2.)
None (you broke even).